KUNS-1446
HE(TH) 97/09

Gauge Freedom in Chiral Gauge Theory with Vacuum Overlap – Four-dimensional case

Yoshio Kikukawa*
Department of Physics and Astronomy, Rutgers University†
Piscataway, NJ 08855-0845

February 1, 2008

Abstract

Dynamical nature of the gauge degree of freedom and its effect to fermion spectrum are studied for four-dimensional nonabelian chiral gauge theory in the vacuum overlap formulation. The covariant gauge fixing term and the Faddeev-Popov determinant are introduced by hand as a weight for the gauge average. At $\beta = \infty$, as noticed by Hata some time ago, the model is renormalizable at one-loop and the gauge fixing term turns into an asymptotically free self-coupling of the gauge freedom, even in the presence of the gauge symmetry breaking of the complex phase of chiral determinant. The severe infrared divergence occurs in the perturbation expansion and it prevents local order parameters from emerging. The Foerster-Nielsen-Ninomiya mechanism works in this perturbative framework: the small explicit gauge symmetry breaking term does not affect the above infrared structure. Based on these dynamical features, which is quite similar to the two-dimensional nonlinear sigma model, we assume that the global gauge symmetry does not break spontaneously and the gauge freedom acquires mass dynamically. The asymptotic freedom allows us to tame the gauge fluctuation by approaching the critical point of the gauge freedom without spoiling its disordered nature. Then it is argued that the disordered gauge freedom does not necessarily cause the massless chiral state in the (waveguide) boundary correlation function and that the entire fermion spectrum can be chiral.

*e-mail address: kikukawa@physics.rutgers.edu, kikukawa@gauge.scphys.kyoto-u.ac.jp
†On leave of absence from: Department of Physics, Kyoto University, Kyoto 606-01, Japan

1 Introduction

It has been one of the most important issues to clarify the dynamical behavior of the gauge freedom and its effect to the fermion spectrum for various proposals of lattice chiral gauge theory[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. In this article, we discuss this issue in the context of the vacuum overlap formulation[16].

In the vacuum overlap formulation of a generic chiral gauge theory, gauge symmetry is explicitly broken by the complex phase of fermion determinant. In order to restore the gauge invariance, gauge average —the integration along gauge orbit— is invoked. Then, what is required for the dynamical nature of the gauge freedom at $\beta = \infty$ (pure gauge limit) is that the global gauge symmetry is not broken spontaneously and all the bosonic field of the gauge freedom could be heavy compared to a typical mass scale of the theory so as to decouple from physical spectrum[18, 1, 2, 3, 7, 16]. For this mechanism of the gauge symmetry restoration to work, the gauge symmetry breaking must be "small" so that it does not spoil the disordered nature of the gauge freedom and it keeps the correlation length of the gauge freedom in the order of the lattice spacing.

In the previous paper on the two-dimensional nonabelian gauge theory[19], we have discussed that a certain gauge symmetry breaking term turns out to be the asymptotically free self-coupling of the gauge freedom and it can be made "large" without spoiling the disordered nature of the gauge freedom. Using this technique to deform the original theory, we have argued that the disordered gauge degree of freedom does not necessarily cause the massless chiral state in the (waveguide) boundary correlation function[12]. We have also argued that the decoupling of the gauge freedom can occur as the self-coupling is removed, provided that the IR fixed point due to the Wess-Zumino-Witten term is absent by anomaly cancellation.

Our objective in this paper is to extend the above argument to nonabelian chiral gauge theory in *four-dimensions*. For this purpose, we reminds us the following facts noticed by Hata[20] some time ago: *at $\beta = \infty$, the covariant gauge fixing term turns into an asymptotically free self-coupling of the gauge freedom in terms of gauge parameter and the severe IR divergence, which occurs in the perturbation (spinwave) expansion, prevents local order parameters from emerging.* These dynamical properties are quite similar to the two-dimensional nonlinear sigma model and they suggest that the gauge freedom acquires mass dynamically and the global gauge symmetry does not break spontaneously for the entire region of the gauge parameter. This is the very nature of the gauge freedom which is required for the dynamical restoration of the gauge symmetry in the lattice chiral gauge theory.

With this dynamical picture in mind, we first introduce the covariant gauge fixing term and the Faddeev-Popov determinant by hand into the lattice chiral gauge theory defined by the vacuum overlap *as a weight for the gauge average.* Then we examine the pure gauge model at $\beta = \infty$ by the perturbation expansion *in the presence of the gauge symmetry breaking terms of the complex phase of chiral determinants.* We will find at one-loop that the model is actually

renormalizable and the self-coupling is asymptotically free. We will see that the IR structure remains same and it prevents local order parameters from emerging. We will also see that this IR structure is not affected by the small perturbation of the gauge symmetry breaking term linear in the link variables, in accord with the Foerster-Nielsen-Ninomiya mechanism. Based on these observations, we will assume that the gauge freedom acquires mass dynamically and the global gauge symmetry does not break spontaneously in the pure gauge limit.

With this assumption, we can make an argument just as in the case of the two-dimensional nonabelian theory. Namely, the asymptotic freedom allows us to tame the gauge fluctuation by approaching the critical point of the gauge freedom without spoiling its disordered nature. There we will show by the spinwave approximation that the spectrum in the *invariant boundary correlation function* has mass gap of the order of the lattice cutoff and it survives the quantum correction due to the gauge fluctuation. There is no symmetry against the spectrum mass gap. Since the overlap correlation function does not depend on the gauge freedom and does show the chiral spectrum[16], the above fact means that the entire fermion spectrum is chiral.

We will not discuss here how the decoupling of the gauge freedom could occur as the self-coupling is removed, because we do not know yet much about the non-perturbative dynamics of the pure gauge model. We will leave this issue for future study. The lattice formulation of the four-dimensional pure gauge model and the possibility to examine it by the Monte Carlo simulation will be discussed elsewhere[21].

A few comments are in order. The covariant gauge fixing term and the Faddeev-Popov determinant (Faddeev-Popov ghost fields) are introduced in the gauge fixing approach for lattice chiral gauge theory of the Rome group[22]. There *the restoration of the BRST invariance* is attempted non-perturbatively by adding and tuning gauge non-invariant counter terms. Since the chiral gauge symmetry is explicitly broken by the lattice fermion regularization, the addition of the gauge fixing term does not mean the gauge fixing. The integration over the compact gauge link variables means that we are still invoking the gauge average. What is meant by the gauge fixing approach is *to decouple the gauge degree of freedom kinematically with the help of the BRST invariance* which could be restored by the fine tuning of the gauge non-invariant counter terms[1]. In this approach, the global gauge symmetry does not need to be realized linearly (symmetrically) and the gauge degree of freedom does not need to be disordered.

In this spirit of the gauge fixing approach, the dynamics of the gauge degree of freedom has been examined by Shamir and Golterman in the model with the covariant-type gauge fixing term[24, 25]. One of the points on which the authors put a stress, among others, is that *the continuum limit should be taken inside the broken phase, because of the species doubling problem in the symmetric phase.* In the broken phase, both the fermions (including species doublers) and the gauge boson acquire masses proportional to the vacuum expectation value of the gauge

[1] It is not yet clear how this non-perturbative restoration of the BRST invariance could overcome the difficulty of the manifest BRST invariant formulation of lattice gauge theory due to Gribov copy noticed by H. Neuberger[23].

freedom (Higgs field). Accordingly, the main concern in the analysis is to seek the critical points inside the broken phase at which the gauge boson becomes massless while keeping the species doublers heavy. The authors identified the boundary between the two broken phases, the ferromagnetic phase(FM) and the helicoidal-ferromagnetic phase(FMD), as such critical points. The latter is characterized by the *vectorial local order parameter which breaks spontaneously the translation and rotation invariance*. Tuning the bare parameters towards this boundary is regarded as the first step to restore the BRST invariance.

Although we also introduce the covariant gauge fixing term and the Faddeev-Popov determinant as a weight for the gauge average, we do not try to recover the BRST invariance. Rather, we seek the possibility to keep the gauge symmetry breaking as small as possible and to restore the gauge symmetry at low energy by the mechanism of Foerster-Nielsen-Ninomiya in the case of the compact nonabelian gauge group. This requires that the global gauge symmetry is not broken spontaneously and that the gauge degree of freedom is kept disordered and heavy compared to the physical scale. (For this very requirement, we admit certain fine tuning of the gauge non-invariant counter terms.) In the overlap formulation, the gauge symmetry breaking is reduced into only the parity odd (imaginary) terms of the chiral determinant. A possible dynamical but perturbative picture for the gauge symmetry restoration for the nonabelian theory is given by Hata, as described above. In this setting, we try to show that *the disordered nature of the gauge freedom does not contradict with the chiral fermion spectrum* in the overlap formulation. We invoke the spinwave approximation in order to discuss *the symmetric phase dynamics*, which validity has been argued in several interesting contexts of the two-dimensional quantum field theories. In order to investigate this dynamical possibility fully, we need the non-perturbative study of the model which could incorporate the effect of the complex action. We hope that the perturbative analysis presented here will give us a physical and dynamical picture how the lattice chiral gauge theory could emerge through the gauge average, and will motivate us to do such non-perturbative study of the dynamics of the gauge symmetry restoration.

This paper is organized as follows. In section 2, we define with vacuum overlap a generic four-dimensional $SU(N)$ chiral gauge theory. Then, we clarify its structure in the pure gauge limit and introduce the covariant gauge fixing term and the Faddeev-Popov determinant as a weight for the gauge average. In section 3, we discuss the dynamics of the nonabelian pure gauge model in the presence of the gauge symmetry breaking terms in the complex phase of the fermion determinant. In section 4, we give a perturbative analysis of the Foerster-Nielsen-Ninomiya mechanism in the context of the pure gauge model. In section 5, we introduce the boundary correlation functions and examine them near the critical point. In section 6, we summarize and discuss our result.

2 Pure gauge limit with covariant gauge fixing term

2.1 Four-dimensional $SU(N)$ Chiral Gauge Theory

Let us consider the four-dimensional $SU(N)$ chiral gauge theory with left-handed Weyl fermions in an anomaly free representation[2]:

$$\begin{aligned}\sum_{rep.} \mathrm{Tr}\left(T^a\left\{T^b,T^c\right)\right\}[r] &= \sum_{rep.} A[r]\times\frac{1}{2}\,d^{abc}[\text{fundamental rep.}]\\ &= 0.\end{aligned}\tag{2.5}$$

The partition function of the chiral gauge theory is given by the following formula in the vacuum overlap formulation[16][3].

$$Z=\int[dU]\exp\left(-\beta S_G\right)\prod_{rep.}\left(\frac{\langle+|v+\rangle}{|\langle+|v+\rangle|}\langle v+|v-\rangle\frac{\langle v-|-\rangle}{|\langle v-|-\rangle|}\right).\tag{2.6}$$

In this formula, $|v+\rangle$ and $|v-\rangle$ are the vacua of the second-quantized Hamiltonians of the five-dimensional Wilson fermion with positive and negative bare masses, respectively.

$$\hat{H}_\pm=\hat{a}^{\dagger}_{n\alpha}{}^{i}H_{\pm\,n\alpha,m\beta}{}_i^j\,\hat{a}_{m\beta j},\tag{2.7}$$

$$\begin{aligned}H_{\pm\,n\alpha,m\beta}{}_i^j &= \begin{pmatrix} B_{nm}{}_i^j\pm m_0\delta_{nm}\delta_i^j & C_{nm}{}_i^j\\ C^{\dagger}_{nm}{}_i^j & -B_{nm}{}_i^j\mp m_0\delta_{nm}\delta_i^j\end{pmatrix}, &(2.8)\\ B_{nm}{}_i^j &= \frac{1}{2}\sum_\mu\left(2\delta_{n,m}\delta_i^j-\delta_{n+\hat\mu,m}U_{n\mu}{}_i^j-\delta_{n,m+\hat\mu}U^{\dagger}_{m\mu}{}_i^j\right), &(2.9)\\ C_{nm}{}_i^j &= \frac{1}{2}\sum_\mu\sigma_\mu\left(\delta_{n+\hat\mu,m}U_{n\mu}{}_i^j-\delta_{n,m+\hat\mu}U^{\dagger}_{m\mu}{}_i^j\right). &(2.10)\end{aligned}$$

[2] Our convention for the $SU(N)$ group generators is as follows:

$$\begin{aligned}\left[T^a,T^b\right] &= if^{abc}T^c, &(2.1)\\ \mathrm{Tr}\left(T^aT^b\right)[r] &= \frac{1}{2}\delta^{ab}\,k[r], &(2.2)\\ \mathrm{Tr}\left(T^a\left\{T^b,T^c\right\}\right)[r] &= \frac{1}{2}\,k[r]\,d^{abc}[r]=\frac{1}{2}d^{abc}[\text{fundamental}]\,A[r], &(2.3)\\ \sum_a T^aT^a &= C_2[r]\,\mathbb{1}[r],\quad C_2[r]=\frac{N^2-1}{2\dim[r]}k[r]. &(2.4)\end{aligned}$$

They are normalized by the fundamental representation so that $k[r]=1$ and $A[r]=1$ for it.

[3] We may start from the partition function with the manifest local gauge invariance, by introducing the explicit integration over the gauge degree of freedom. We will discuss about this alternative gauge invariant formulation and the various gauge fixings in the discussion in relation to the Wilson-Yukawa model.

$|+\rangle$ and $|-\rangle$ are corresponding free vacua. The Wigner-Brillouin phase convention is explicitly implemented by the overlaps of vacua with the same signature of mass. $\prod_{rep.}$ stands for the product over all Weyl fermion multiplets in the anomaly free representation. S_G is the gauge action.

In the vanishing gauge coupling limit $\beta = \infty$, the gauge link variable is given in the pure gauge form:

$$U_{n\mu} = g_n g^{\dagger}_{n+\mu}, \quad g_n \in SU(N). \tag{2.11}$$

Then the model describes the gauge degree of freedom coupled to fermion through gauge non-invariant piece of complex phase of chiral determinants.

$$Z \quad = \quad \int [dg] \prod_{rep.} \left(\frac{\langle +|\hat{G}|+\rangle}{|\langle +|\hat{G}|+\rangle|} \langle +|-\rangle \frac{\langle -|\hat{G}^{\dagger}|-\rangle}{|\langle -|\hat{G}^{\dagger}|-\rangle|} \right). \tag{2.12}$$

$\hat{G}$ is the operator of the gauge transformation given by:

$$\hat{G} = \exp\left(\hat{a}^{\dagger i}_n \{\log g_n\}_i{}^j \hat{a}_{nj} \right). \tag{2.13}$$

As we can see from this pure gauge limit of the original theory, the gauge average is invoked without any weight for the gauge freedom except for the complex phase of the chiral determinant. This way of the gauge average well keep the disordered nature of the gauge freedom and keep its correlation length within the order of the lattice spacing.

2.2 Covariant gauge fixing term as a weight of gauge average

In order to examine the dynamical effect of the gauge average closely, it is desirable to have control over the fluctuation of the gauge freedom[19, 26]. For this purpose, we add to the original theory by hand the covariant gauge fixing term and the Faddeev-Popov determinant (ghost field) *as a weight for the gauge average*:

$$\begin{aligned}
&\exp\left(-\sum_{n,a} \frac{1}{2\alpha} \left(\sum_{\mu} \bar{\nabla}_{\mu} \hat{A}^a_{n\mu} \right)^2 \right) \det\left(\hat{M}^{ab}_{nm}\left[U_{n\mu}\right] \right) \\
&= \int [dc^a d\bar{c}^a] \exp\left(-\sum_{n,a} \frac{1}{2\alpha} \left(\sum_{\mu} \bar{\nabla}_{\mu} \hat{A}^a_{n\mu} \right)^2 - \sum_{nm,ab} \bar{c}^a_n \hat{M}^{ab}_{nm} c^b_m \right),
\end{aligned} \tag{2.14}$$

where

$$\hat{A}_{n\mu} = \frac{1}{2i}\left(U_{n\mu} - U^{\dagger}_{n\mu} \right) - \frac{1}{N} \mathbb{1} \, \mathrm{Tr} \, \frac{1}{2i}\left(U_{n\mu} - U^{\dagger}_{n\mu} \right), \tag{2.15}$$

and

$$
\begin{aligned}
\hat{M}^{ab}_{nm}\left[U_{n\mu}\right] &= \sum_{\mu}\Big[\Big\{\hat{E}^{-1}_{ab}(U_{n\mu})\delta_{nm} - \hat{E}^{-1}_{ba}(U_{n\mu})\delta_{n+\hat{\mu},m}\Big\} \\
&\qquad - \Big\{\hat{E}^{-1}_{ab}(U_{n-\hat{\mu},\mu})\delta_{n-\hat{\mu},m} - \hat{E}^{-1}_{ba}(U_{n-\hat{\mu},\mu})\delta_{n,m}\Big\}\Big], \qquad (2.16) \\
\hat{E}^{-1}_{ab}(U_{n,\mu}) &= \mathrm{Tr}\left(T^a T^b U_{n\mu} + T^b T^a U^{\dagger}_{n\mu}\right). \qquad (2.17)
\end{aligned}
$$

This covariant gauge fixing term[27] is formally based on the lattice Landau gauge

$$
\bar{\nabla}_\mu \hat{A}_{n\mu} = 0, \qquad (2.18)
$$

which minimizes locally the "functional norm" of the link variables on the given gauge orbit,

$$
F[{}^{g}U_{n\mu}] = 1 - \frac{1}{NDV}\sum_{n\mu}\mathrm{Tr}\,{}^{g}U_{n\mu}. \qquad (2.19)
$$

The structure of the Faddeev-Popov operator Eq. (2.16) has been examined extensively in [28].

It should be noted that since the gauge symmetry is explicitly broken in the vacuum overlap formulation, the addition of the gauge fixing term does not mean the gauge fixing. The integration over the compact gauge link variables means that we are still invoking the gauge average to restore the gauge symmetry. The role of the gauge fixing term is to modify the weight for the gauge degree of freedom in the gauge average.

Accordingly, the BRST invariance does not hold in this model. The BRST transformation can be defined in this model as follows[29]:

$$
\begin{aligned}
\boldsymbol{\delta}_B \hat{A}^a_{n\mu} &= \Big\{\hat{E}^{-1}_{ab}(U_{n\mu})\,c^b_n - \hat{E}^{-1}_{ba}(U_{n\mu})\,c^b_{n+\hat{\mu}}\Big\} \qquad (2.20) \\
\boldsymbol{\delta}_B c^a_n &= -\frac{1}{2} f_{abc}\, c^b_n\, c^c_n, \qquad (2.21) \\
\boldsymbol{\delta}_B \bar{c}^a_n &= -\frac{1}{\alpha}\sum_{\mu}\bar{\nabla}_\mu \hat{A}^a_{n\mu}. \qquad (2.22)
\end{aligned}
$$

The nilpotency holds on the gauge potential, $\hat{A}^a_{n\mu}$:

$$
\boldsymbol{\delta}^2_B \hat{A}^a_{n\mu} = 0. \qquad (2.23)
$$

Then the gauge fixing term and the Faddeev-Popov ghost action are BRST invariant. The functional integral measure of the gauge field and ghost and anti-ghost fields are also BRST invariant. Only the complex phases of the chiral determinants are BRST non-invariant, as they should be. If the BRST invariance would exist on the lattice, the partition function must vanish as shown by Neuberger[23]. The failure of the manifestly BRST invariant formulation of the lattice gauge theory is due to the following reason. On a finite lattice and for

a compact gauge group, there are even number of solutions for the gauge fixing condition, namely, the Gribov copies. Because of the compactness of the gauge configuration space, half of them have the positive Faddeev-Popov determinants and half of them have the negative ones. They cancel each other to result in the vanishing partition function. In the present case, however, the gauge symmetry breaking terms in the complex phase of the chiral determinant distinguish these Gribov copies[30] and resolve the degeneracy. They are irrelevant in the naive continuum limit, but they actually prevent the lattice Gribov copy (at $\beta = \infty$) from being a classical solution of the equation of motion, as we will see in the following section 3.1.

2.3 Lattice $SU(N)$ pure gauge model with chiral fermions

In the pure gauge limit, the model reduces to the $SU(N)$ pure gauge model[20] in the lattice regularization which also couples to anomaly-free chiral fermions through the gauge non-invariant piece of complex phase of chiral determinant:

$$\begin{aligned} Z &= \int [dg] \exp\left(-\sum_{n,a} \frac{1}{2\alpha} \left(\sum_{\mu} \bar{\nabla}_\mu \hat{A}^a_{n\mu} \right)^2 \right) \det\left(\hat{M}^{ab}_{nm} \left[g_n g^\dagger_{n+\hat{\mu}} \right] \right) \times \\ & \qquad \prod_{rep.} \left(\frac{\langle + | \hat{G} | + \rangle}{|\langle + | \hat{G} | + \rangle|} \langle + | - \rangle \frac{\langle - | \hat{G}^\dagger | - \rangle}{|\langle - | \hat{G}^\dagger | - \rangle|} \right) \\ &\equiv \int d\mu[g;\alpha], \end{aligned} \tag{2.24}$$

where

$$\hat{A}_{n\mu} = \frac{1}{2i} \left(g_n g^\dagger_{n+\hat{\mu}} - g_{n+\hat{\mu}} g^\dagger_n \right) - \frac{1}{N} \mathbb{1} \operatorname{Tr} \frac{1}{2i} \left(g_n g^\dagger_{n+\hat{\mu}} - g_{n+\hat{\mu}} g^\dagger_n \right). \tag{2.25}$$

Including the imaginary action, the functional integral measure of the gauge freedom is denoted by $d\mu[g;\alpha]$.

This model is invariant under two global $SU(N)$ transformations acting on the field of the gauge freedom. The first one is the global remnant of the gauge transformation:

$$SU(N)_C : \qquad {g_n}_i^j \longrightarrow {g_0}_i^k {g_n}_k^j. \tag{2.26}$$

The second one comes from the arbitrariness of choice of pure gauge variable g_n:

$$SU(N)_H : \qquad {g_n}_i^j \longrightarrow {g_n}_i^k {h^\dagger}_k^j. \tag{2.27}$$

They defines the chiral transformation of $G = SU(N)_L \times SU(N)_R = SU(N)_C \times SU(N)_H$ and the model is symmetric under this chiral transformation.

We refer the imaginary part of the action of the gauge freedom induced from the fermion determinant as the Wess-Zumino-Witten term, although the actual Wess-Zumino-Witten terms are canceled among the fermions. We denote it by

$\Delta\Gamma_{WZW}$:

$$e^{i\Delta\Gamma_{WZW}[g]} \equiv \prod_{rep.} \left(\frac{\langle + | \hat{G} | + \rangle}{|\langle + | \hat{G} | + \rangle|} \frac{\langle - | \hat{G}^\dagger | - \rangle}{|\langle - | \hat{G}^\dagger | - \rangle|} \right). \tag{2.28}$$

The explicit formula of the Wess-Zumino-Witten term is given by

$$\begin{aligned} & i\Delta\Gamma_{WZW}[g] \\ &= \sum_{rep.} \Bigg\{ i \mathrm{Im} \mathrm{Tr}\, \mathrm{Ln} \left[\sum_m v_+^\dagger(p,s) e^{-ipm} \left(g_{m}{}_i^j \right) e^{iqm} v_+(q,s') \right] \\ &\qquad + i \mathrm{Im} \mathrm{Tr}\, \mathrm{Ln} \left[\sum_m v_-^\dagger(p,s) e^{-ipm} \left(g^\dagger_{m}{}_i^j \right) e^{iqm} v_-(q,s') \right] \Bigg\}. \end{aligned} \tag{2.29}$$

Here $v_+(p,s)$ and $v_-(p,s)$ are negative-energy eigenvectors of the free Hamiltonians with positive and negative masses, respectively.

Note also that even in the limit $\alpha \to \infty$ where the covariant gauge fixing term vanishes, the modified model does not completely reduce to the original model: the ghost and anti-ghost fields remain to couple to the gauge freedom.

3 Pure gauge dynamics with covariant gauge fixing term

In this section, following the argument given by Hata in the continuum limit, we examine the dynamical effect of the gauge degree of freedom by the perturbation expansion (spinwave approximation) in terms of λ ($\lambda^2 \equiv \alpha$). We first invoke the background field method[31] in order to examine the quantum effect of the gauge freedom and the renormalization due to it, including the contributions from the the Wess-Zumino-Witten term. Then we examine the realization of the global gauge symmetry at $\beta = \infty$.

3.1 Classical solution for the equation of motion and Gribov copy

We consider the dynamical degree of freedom of the gauge freedom, the ghost and anti-ghost fields as the fluctuations from the classical configurations which satisfies the equation of motion:

$$g_n \rightarrow \exp(i\lambda\pi_n)\, g_n, \tag{3.1}$$

$$c_n^a \rightarrow c_n^a + \xi_n^a, \tag{3.2}$$

$$\bar{c}_n^a \rightarrow \bar{c}_n^a + \bar{\xi}_n^a. \tag{3.3}$$

By substituting the above decompositions of classical and quantum components into the action, it is expanded up to the quadratic terms of the fluctuations

π, ξ and $\bar{\xi}$. And then by performing the Gaussian functional integration of the fluctuations, we evaluate the quantum correction to the classical action at one-loop.

The linear terms in the fluctuation, π, ξ and $\bar{\xi}$ give the equations of motion for the classical configurations g, c and $\bar{c}$. For the gauge freedom, it is given as

$$\begin{aligned} - & \frac{1}{\alpha}\sum_{n\mu}\bar{\nabla}_\mu \hat{A}^a_{n\mu}\hat{M}^{ac}_{nm}\left[g_n g^\dagger_{n+\hat{\mu}}\right] \\ + & \sum_{n\mu}\Big[\Big\{\bar{c}^a_n F^{abc}_{nm}\left[g_n g^\dagger_{n+\hat{\mu}}\right]c^b_n - \bar{c}^a_n F^{bac}_{nm}\left[g_n g^\dagger_{n+\hat{\mu}}\right]c^b_{n+\hat{\mu}}\Big\} \\ & \quad -\left\{\bar{c}^a_n F^{abc}_{n-\hat{\mu},m}\left[g_{n-\hat{\mu}} g^\dagger_{n}\right]c^b_{n-\hat{\mu}} - \bar{c}^a_n F^{bac}_{nm}\left[g_{n-\hat{\mu}} g^\dagger_{n}\right]c^b_{n}\right\}\Big] \\ + & \frac{i}{2}\sum_{rep.}\Big\{\mathrm{Tr}\left(S^v_+[g^\dagger](n,m)T^c g_m\right) + \mathrm{Tr}\left(S^v_+[g](n,m)g^\dagger_m T^c\right) \\ & \quad -\mathrm{Tr}\left(S^v_-[g^\dagger](n,m)T^c g_m\right) - \mathrm{Tr}\left(S^v_-[g](n,m)g^\dagger_m T^c\right)\Big\} = 0, \end{aligned} \tag{3.4}$$

where

$$\begin{aligned} F^{abc}_{nm}\left[g_n g^\dagger_{n+\hat{\mu}}\right] &= i\mathrm{Tr}\left(T^aT^bT^c g_n g^\dagger_{n+\hat{\mu}} - T^cT^bT^a g_{n+\hat{\mu}} g^\dagger_n\right)\delta_{nm} \\ &- i\mathrm{Tr}\left(T^cT^aT^b g_n g^\dagger_{n+\hat{\mu}} - T^bT^aT^c g_{n+\hat{\mu}} g^\dagger_n\right)\delta_{n+\hat{\mu},m}, \end{aligned} \tag{3.5}$$

and

$$\begin{aligned} & S^v_\pm[g](n,m)^j_i \\ & \quad \equiv \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\, e^{ipn}e^{-iqm}\times \\ & \qquad v_\pm(p,s)\left[v^\dagger_\pm(q,s')e^{-iqr}\left(g^{\dagger i}_{rj}\right)e^{ipr}v_\pm(p,s)\right]^{-1}_{(p,s,i)(q,s',j)} v^\dagger_\pm(q,s'). \end{aligned} \tag{3.6}$$

Note that the last term in the l.h.s. of the equation of motion is imaginary, which comes from the imaginary part of the action, $i\Delta\Gamma_{WZW}[g]$. Note that this term depends non-locally on the field variable of the gauge freedom. This non-locality is an inevitable feature expected for any gauge non-invariant regularization of fermion. We should note that this classical equation of motion carries the information about the anomaly cancellation. In anomaly-free theories, this imaginary term is irrelevant in the continuum limit which can be taken for the slowly varying solution. Otherwise, it contains the finite term which corresponds to the variation of the Wess-Zumino-Witten term in the continuum limit.

The simplest and trivial solution for the equation of motion is given by

$$g_n = \mathbb{1}, \quad c^a_n = 0, \quad \bar{c}^a_n = 0. \tag{3.7}$$

It solves the imaginary part simply because the matrix to be inverted in Eq. (3.6) become simple products of the unit matrixes both in the group space and in the

momentum and spinor spaces:

$$\left[v_{\pm}^{\dagger}(q,s')e^{-iqr}\left(g_{rj}^{\dagger i}\right)e^{ipr}v_{\pm}(p,s)\right]=(2\pi)^4\delta^4(q-p)\delta_{ss'}\delta_j^i. \tag{3.8}$$

Then the trace over the group indices vanishes. Moreover, the dependence on the signatures of the masses $\pm m_0$ drop off in each contribution and they can cancel among the contributions of the positive and negative masses. (This reason applies to the case of the U(1) gauge theory.)

It is interesting to note the fact that the lattice Gribov copies (at $\beta=\infty$) in the usual sense, which solve the real part of the equation of motion, does not necessarily solve the imaginary part of the equation. To see this, let us consider the simple example of the lattice Gribov copy[32, 33, 24] given as

$$g_n=\left\{\begin{array}{cc}\mathbb{1} & (n\neq n_0)\\ \mathrm{diag}(-1,-1,1,\ldots,1) & (n=n_0)\end{array}\right. \qquad c_n^a=0, \quad \bar{c}_n^a=0, \tag{3.9}$$

In this case, the group space and the momentum and spinor spaces are mixed in the matrix to be inverted. Then the trace over the group indices does not vanish in general. The dependence on the signatures of the masses $\pm m_0$ does not drop off either and the cancellation hardly occurs. Therefore, this example of the Gribov copy does not solve the equation of motion. (The rigorous proof is beyond the scope of this paper. It is not difficult to check numerically on finite lattice that the imaginary part does not vanish for the given lattice Gribov copy.) The lattice Gribov copies which do not solve the imaginary part of the equation of motion are no more the stationary point of the total action and are suppressed by the quantum fluctuation around them in the gauge average. This is the way how the irrelevant gauge symmetry breaking terms in the complex phase of chiral determinant can resolve the degeneracy due to the lattice Gribov copies. Note that we do not claim here that all the Gribov copies should be suppressed. Especially, not for the continuum Gribov copies. Rather we point out a possible mechanism to suppress the lattice Gribov copies in the case of the lattice chiral gauge theory.

3.2 Gaussian gauge fluctuation and Infrared singularity

Let us assume that a classical solution for the equations of motion of g, c and $\bar{c}$ is given. We parameterize the classical solution g_n as[4]

$$g_n g_{n+\hat{\mu}}^{\dagger}=\exp\left(iA_{n\mu}\right). \tag{3.11}$$

For the technical reason to extract the local operators of the background fields, we also assume that this solution is sufficiently slowly varying in the lattice unit:

$$\nabla_\nu A_{n\mu} << A_{n\mu}. \tag{3.12}$$

[4] The relation to the pure gauge vector potential $\hat{A}_{n\mu}$ of Eq. (2.25) is given by

$$\hat{A}_{n\mu}^a=2\mathrm{Tr}\left\{T^a\sin A_{n\mu}\right\}. \tag{3.10}$$

Then we expand the action in terms of the fluctuations π, ξ, $\bar{\xi}$ up to quadratic terms as

$$\begin{aligned} S[\exp(i\pi)g, c+\xi, \bar{c}+\bar{\xi}] &= S[g,c,\bar{c}] + S_0[\pi^2/\xi,\bar{\xi}] \\ &+ S_1[\pi^2/\xi,\bar{\xi};\sin A] + S_{1c}[\pi,\xi/\bar{\xi};g,c/\bar{c}] \\ &+ S_2[\pi^2/\xi,\bar{\xi};\sin^2 A, \cos A - 1] + S_{2c}[\pi^2;g,c,\bar{c}] \\ &+ i\,\Delta S_{WZW}[\pi^2;g]. \end{aligned} \tag{3.13}$$

The explicit formula of these terms are given in the appendix A.

S_0 is the free action of the fluctuations π, ξ and $\bar{\xi}$.

$$S_0[\pi^2/\xi,\bar{\xi}] = -\sum_{n,a} \frac{1}{2}\left(\nabla^2 \pi_n^a\right)^2 + \sum_{n,a} \bar{\xi}_n^a \nabla^2 \xi_n^a. \tag{3.14}$$

This gives a massless dipole propagator for π[20]:

$$\langle \pi_n^a \pi_m^b \rangle = \delta^{ab} \int \frac{d^4p}{(2\pi)^4} e^{ip(n-m)} \frac{1}{\left(\sum_\mu 4\sin^2\frac{p_\mu}{2}\right)^2} \equiv \delta^{ab} G(n-m). \tag{3.15}$$

This is infrared(IR) divergent in four-dimensions and we need a certain IR regularization in the perturbation expansion. For this, we add the mass term for π,

$$-\sum_{na} \frac{1}{2}\,\mu_0^4 \pi_n^a \pi_n^a. \tag{3.16}$$

(We denote the dimensionless mass parameter with subscript "0" as μ_0 and the dimensional one without it: $\mu a = \mu_0$.) As for the gauge fixing sector, we may invoke the dimensional regularization, which preserves the BRST invariance of this sector.

$i\Delta S_{WZW}$ is the contribution from the Wess-Zumino-Witten term. According to the decomposition of the classical solution g_n and the quantum fluctuation π_n, we can write the Wess-Zumino-Witten term as

$$\begin{aligned} e^{i\Delta S_{WZW}[\pi;g]} &= e^{i\Delta\Gamma_{WZW}[\exp(i\lambda\pi)g] - i\Delta\Gamma_{WZW}[g]} \\ &= \prod_{rep.} \left(\frac{\langle +|\hat{\Pi}\hat{G}|+\rangle}{|\langle +|\hat{\Pi}\hat{G}|+\rangle|} \frac{\langle -|\hat{G}^\dagger\hat{\Pi}^\dagger|-\rangle}{|\langle -|\hat{G}^\dagger\hat{\Pi}^\dagger|-\rangle|} \Big/ \frac{\langle +|\hat{G}|+\rangle}{|\langle +|\hat{G}|+\rangle|} \frac{\langle -|\hat{G}^\dagger|-\rangle}{|\langle -|\hat{G}^\dagger|-\rangle|} \right) \\ &= \prod_{rep.} \left(\frac{\langle +|\hat{\Pi}|v+\rangle}{|\langle +|\hat{\Pi}|v+\rangle|} \frac{\langle v-|\hat{\Pi}^\dagger|-\rangle}{|\langle v-|\hat{\Pi}^\dagger|-\rangle|} \Big/ \frac{\langle +|v+\rangle}{|\langle +|v+\rangle|} \frac{\langle v-|-\rangle}{|\langle v-|-\rangle|} \right). \end{aligned} \tag{3.17}$$

$\hat{\Pi}$ is the operator of the gauge transformation due to π given by:

$$\hat{\Pi} = \exp\left(\hat{a}_n^{\dagger i}\{i\lambda\pi_n\}_i{}^j \hat{a}_{nj}\right). \tag{3.18}$$

$|v+\rangle$ and $|v-\rangle$ in this case are given by the vacua of the second-quantized Hamiltonians with the pure gauge link variable Eq. (3.11), which consist of the classical solutions g_n. We can evaluate these vacua in the expansion in terms of

$A_{n\mu}$ using the Hamiltonian perturbation theory[16, 34, 35]. Then $i\Delta S_{WZW}$ is obtained in the form

$$i\,\Delta S_{WZW}[\pi; g] = \sum_{k=1}^{\infty} i\,\Delta S_{k\,WZW}[\pi; A^k]. \tag{3.19}$$

This result is further expanded in terms of π. The linear term in π gives the contribution to the equation of motion. (Note that the formula for this contribution given in Eq. (3.4) is evaluated directly from the first expression in Eq. (3.17).) The quadratic term in π is what we need for the one-loop calculation by the background field method. We can see that every vertexes of A_μ's and π's obtained in this expansion of $i\Delta S_{WZW}$ are *local in the unit of the lattice spacing*. This is because they can be regarded as the loop effect of the (five-dimensional) Wilson fermion with the mass $\pm m_0$ of the order of the lattice cutoff.

We can show that the leading and the next-to-leading terms vanish identically in anomaly-free theories. The leading term has a generic form as

$$\begin{aligned} i\Delta S_{1\,WZW}[\pi^2; A] \quad = \quad & \sum_{rep.} \sum_{n,l_1,l_2} \mathrm{Tr}\left\{A_{n\mu}\pi_{l_1}\pi_{l_2}\right\} \times \\ & \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\, e^{ik_1(l_1-m)+ik_2(l_2-m)}\,\Gamma_{1\,WZW\,\mu}(k_1,k_2). \end{aligned} \tag{3.20}$$

Since this imaginary term must be odd under the charge conjugation[16]:

$$A_{n\mu} \longrightarrow -A^*_{n\mu}, \tag{3.21}$$
$$\pi_n \longrightarrow -\pi^*_n, \tag{3.22}$$

it turns out to be proportional to the factor

$$\begin{aligned} \sum_{rep.} \mathrm{Tr}\left\{A_{n\mu}\pi_{l_1}\pi_{l_2} + \pi^*_{l_2}\pi^*_{l_1}A^*_{n\mu}\right\} \quad &= \quad \sum_{rep.} \mathrm{Tr}\left\{A_{n\mu}\left\{\pi_{l_1},\pi_{l_2}\right\}\right\} \\ &= \quad \sum_{rep.} d^{abc}A^a_{n\mu}\pi^b_{l_1}\pi^c_{l_2}, \end{aligned} \tag{3.23}$$

Therefore, it vanishes in anomaly-free theories.

$$i\,\Delta S_{1\,WZW}[\pi^2; A] = 0. \tag{3.24}$$

The next-to-leading term $i\Delta S_{2\,WZW}$ has the following generic form.

$$\begin{aligned} & i\Delta S_{2\,WZW}[\pi^2; A^2] \\ & = \sum_{n,m,l_1,l_2} \mathrm{Tr}\left\{A_{n\mu}A_{m\nu}\pi_{l_1}\pi_{l_2}\right\} \times \\ & \quad \int \frac{d^4p}{(2\pi)^4}\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\, e^{ip(m-n)+ik_1(l_1-n)+ik_2(l_2-n)}\,\Gamma_{2\,WZW\,\mu\nu}(p,k_1,k_2). \end{aligned} \tag{3.25}$$

This term is also odd under the charge conjugation and it turns out to be proportional to the factor

$$\begin{aligned}
&\sum_{rep.} \mathrm{Tr}\left\{A_{n\mu}A_{m\nu}\pi_{l_1}\pi_{l_2} - \pi^*_{l_2}\pi^*_{l_1}A^*_{m\nu}A^*_{n\mu}\right\} \\
&= \sum_{rep.} \mathrm{Tr}\left\{T^aT^bT^cT^d - T^bT^aT^dT^c\right\} A^a_{n\mu}A^b_{m\nu}\pi^c_{l_1}\pi^d_{l_2} \\
&= \sum_{rep} \frac{1}{4}\, k[r]\, \left(if^{abe}d^{cde}[r] + id^{abe}[r]f^{cde}\right) A^a_{n\mu}A^b_{m\nu}\pi^c_{l_1}\pi^d_{l_2}, \qquad (3.26)
\end{aligned}$$

where we have used the identity

$$T^aT^b = \frac{1}{2}\, if^{abe}T^e + \frac{1}{2}\, d^{abe}[r]T^e + \frac{1}{2}\, k[r]\, \delta^{ab}\, \frac{1}{\dim[r]}\mathbb{1}. \qquad (3.27)$$

Therefore, the next-to-leading term also vanishes for anomaly free theories.

$$i\Delta S_{2\,WZW}[\pi^2; A^2] = 0. \qquad (3.28)$$

As we will see in the next subsection, the fact that the two leading terms of $i\Delta S_{WZW}$ vanish identically[5] is sufficient for the one-loop renormalizability of the pure gauge model. The structure of these two terms are discussed in some detail in the appendix C.

3.3 Quantum effect at one-loop and asymptotic freedom

The quantum correction to the classical action is obtained by performing the Gaussian functional integration with S_0,

$$\exp\left(\Delta S[g, c, \bar{c}]\right) = \int [d\pi d\xi d\bar{\xi}]\, e^{S_0} \sum_{l=0}^{\infty} \frac{1}{l!}\left(S_1 + S_{1c} + S_2 + S_{2c} + iS_{WZW}\right)^l, \qquad (3.29)$$

[5] The third order term does not seem to vanish identically. This term turns out to be proportional to the factor

$$\begin{aligned}
&\sum_{rep.} \mathrm{Tr}\left\{A_{n\mu}A_{m\nu}A_{p\lambda}\pi_{l_1}\pi_{l_2} + \pi^*_{l_2}\pi^*_{l_1}A^*_{p\lambda}A^*_{m\nu}A^*_{n\mu}\right\} \\
&= \sum_{rep.} \mathrm{Tr}\left\{T^aT^bT^cT^dT^e + T^eT^dT^cT^bT^a\right\} A^a_{n\mu}A^b_{m\nu}A^c_{p\lambda}\pi^d_{l_1}\pi^e_{l_2} \\
&= \sum_{rep} \left(\frac{i}{8}k[r]\left(if^{abf}d^{cgf} + id^{abf}f^{cgf}\right) f^{deg}\right. \\
&\quad \left. + \frac{1}{8}k[r]\left(2\frac{k[r]}{\dim[r]}\delta^{ab}\delta^{cg} - f^{abf}f^{cgf} + d^{abf}d^{cgf}\right) d^{deg} + \frac{k[r]^2}{4\dim[r]}d^{abc}\delta^{de}\right) \times \\
&\qquad A^a_{n\mu}A^b_{m\nu}A^c_{p\lambda}\pi^d_{l_1}\pi^e_{l_2}.
\end{aligned}$$

We can see that the terms which are not linear in the anomaly coefficient $k[r]\, d^{abc}[r]$ appear in this order.

which can be obtained in the expansion in terms of A_μ, c and $\bar{c}$,

$$\Delta S[g,c,\bar{c}] = \sum_{j=1} \Delta S_j[A^j] + \sum_{k,l=1} \Delta S_{k,l}[A^k,(c,\bar{c})^l]. \tag{3.30}$$

From each one-loop contribution, the local operators which can be written in terms of the pure gauge vector potential A_μ and the ghost and anti-ghost fields, c and $\bar{c}$, should be extracted. The possible local operators of dimensions less than five can be listed up based on the global $SU(N)$ gauge symmetry, the hyper-cubic symmetry, the CP invariance, the ghost number conservation and the anti-ghost shift symmetry[22]. The complete list for the $SU(N)$ gauge group is given in the appendix B. They are denoted by $\mathcal{O}_k\,(k=0,\cdots,13)$. Among them, only $\mathcal{O}_0$ is the BRST invariant combination. Using these operators, the quantum correction can be expressed formally in the form:

$$\Delta S[g,c,\bar{c}] = \sum_k c_k \mathcal{O}_k[A,c,\bar{c}] + \cdots. \tag{3.31}$$

Up to the fourth order in A_μ, c and $\bar{c}$, we obtain

$$\begin{aligned}
\Delta S_1[A] &= \langle S_1[A]\rangle_0, && (3.32)\\
\Delta S_2[A^2] &= \langle S_2[A^2]\rangle_0 + \frac{1}{2!}\langle S_1[A]^2\rangle_0, && (3.33)\\
\Delta S_{0,1}[(c,\bar{c})] &= \langle S_{2c}[(c,\bar{c})]\rangle_0 + \frac{1}{2}\langle S_{1c}[c/\bar{c}]^2\rangle_0, && (3.34)\\
\Delta S_{1,1}[A,(c,\bar{c})] &= \langle S_{2c}[A,(c,\bar{c})]\rangle_0 + \langle S_{2c}[(c,\bar{c})]\,S_1[A]\rangle_0 \\
&\quad + \langle S_{1c}[c/\bar{c}]\,S_{1c}[A,c/\bar{c}]\rangle_0, \\
& && (3.35)\\
\Delta S_3[A^3] &= \langle S_1[A^3]\rangle_0 + \langle S_1[A]S_2[A^2]\rangle_0 + \frac{1}{3!}\langle S_1[A]^3\rangle_0 \\
&\quad + \langle iS_{3\,WZW}[A^3]\rangle_0, && (3.36)\\
\Delta S_4[A^4] &= \langle S_2[A^4]\rangle_0 + \langle S_1[A]S_1[A^3]\rangle_0 + \frac{1}{2!}\langle S_2[A^2]^2\rangle_0 \\
&\quad + \frac{1}{2}\langle S_1[A]^2 S_2[A^2]\rangle_0 + \frac{1}{4!}\langle S_1[A]^4\rangle_0 \\
&\quad + \langle iS_{4\,WZW}[A^4]\rangle_0 + \langle iS_{3\,WZW}[A^2]S_1[A]\rangle_0. \\
& && (3.37)
\end{aligned}$$

Here $\langle\,\rangle_0$ stands for the connected part of the expectation value evaluated with the Gaussian weight S_0. We have taken account of the fact that $\bar{c}$ is always associated with the derivative ∇_μ because of the invariance of the original action under the shift symmetry:

$$\bar{c}_n \longrightarrow \bar{c}_n + \bar{\epsilon}. \tag{3.38}$$

We have also taken account of the fact that $i\Delta S_{1\,WZW}$ and $i\Delta S_{2\,WZW}$ are identically zero.

We can see in the above expressions that all the contributions from the Wess-Zumino-Witten term are imaginary at this order. To be real, it needs at least two imaginary induced vertexes. However, the imaginary induced vertexes quadratic in π, $i\Delta S_{kWZW}[\pi^2; A^k]$'s, start from the third order in the pure gauge vector potential A_μ. Then it must have the dimensions more than five. On the other hand, the local operators of dimensions less than five, $\mathcal{O}_k$'s, preserve parity and charge conjugation invariance separately and are real. Therefore, there occurs no contribution due to the Wess-Zumino-Witten term to these local operators of dimensions less than five at the one-loop. All contributions come from the gauge fixing sector and they respect the BRST invariance.

The BRST invariant operator of dimensions less than five, which is given by $\mathcal{O}_0$ in the list of the appendix B, is nothing but the classical action:

$$\mathcal{O}_0 = \left[-\frac{1}{2\alpha} \sum_n \left(\bar{\nabla}_\mu A^a_{n\mu} \right)^2 - \sum_n \bar{c}^a_n \bar{\nabla}_\mu \left(-\nabla_\mu c^a_n + \frac{1}{2} f^{abc} (c^b_n + c^b_{n+\hat{\mu}}) A^c_\mu \right) \right]. \tag{3.39}$$

Then it is enough to evaluate the terms up to the order $\mathcal{O}(A^2)$. They are given by Eqs. (3.32), (3.33), (3.34) and (3.35), and are evaluated as follows:

$$\Delta S_1 + \Delta S_2 + \Delta S_{1c} + \Delta S_{2c} \quad = \quad -\lambda^2 \left[N\bar{G} + \left(\frac{N^2-1}{N} \right) [G(0) - G(1)] \right] \mathcal{O}_0. \tag{3.40}$$

where

$$\begin{aligned} \bar{G} &= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \frac{\left(\sum_\mu 4\sin^2 \frac{k_\mu}{2} \right) \left(\sum_\mu \sin^2 k_\mu \right)}{\left[\left(\sum_\mu 4 \sin^2 \frac{k_\mu}{2} \right)^2 + \mu_0^4 \right]^2} \\ &\simeq -\frac{1}{16\pi^2} \ln(\mu_0) + \bar{C} \qquad (\mu_0 << 1). \end{aligned} \tag{3.41}$$

The calculation is described in some detail in the appendix E. This logarithmic divergence is renormalizable into λ:

$$\frac{1}{\lambda^2} + \frac{N}{16\pi^2} \ln(a\mu) = \frac{1}{\lambda_R^2}. \tag{3.42}$$

This proves the one-loop renormalizability of the pure gauge model of the lattice $SU(N)$ chiral gauge theory defined by the vacuum overlap. This one-loop renormalizability can be regarded as an indication of the "smallness" of the gauge symmetry breaking which is caused by the gauge non-invariant definition of the complex phase of chiral determinant[16].

The beta function of λ can be evaluated as

$$\beta(\lambda) = -a \frac{\partial \lambda}{\partial a} = -\frac{N}{32\pi^2} \lambda^3, \tag{3.43}$$

which means that λ is asymptotically free, as noticed by Hata[20].

3.4 IR singularity and unbroken global gauge symmetry

In this subsection, we examine the realization of the global gauge symmetry at $\beta = 0$ in the pure gauge model within the framework of the perturbation theory. As we have seen in Eq. (3.15) in the subsection 3.2, the gauge fluctuation shows severe IR singularity. The local order parameter

$$\left\langle g_{n}{}_{i}^{j} \right\rangle \tag{3.44}$$

suffers from the IR divergence and cannot be regarded to be the physical observable. This situation is very analogous to the two-dimensional chiral $SU(N)$ nonlinear sigma model, as noticed by Hata[20]. This analogy suggests us that we would examine the chiral $SU(N)$ invariant two-point function of the gauge freedom, in which the IR divergence is expected to cancel by the similar mechanism in the two-dimensional case[36]. Based on the one-loop renormalizability of the pure gauge model, which we have observed in the previous section, we can examine it also by the renormalization group method.

The invariant two-point correlation function of g is defined by

$$G_g(n) \quad \equiv \quad \left\langle \frac{1}{N} \mathrm{Tr} \left\{ g_n g_0^\dagger \right\} \right\rangle . \tag{3.45}$$

In the perturbation expansion in terms of λ, it is given at the leading order as

$$G_g(n) = 1 + \lambda^2 \left(\frac{N^2 - 1}{2N} \right) \left[G(n) - G(0) \right] , \tag{3.46}$$

where

$$\begin{aligned} \left[G(n) - G(0) \right] \quad &\equiv \quad \int \frac{d^4 p}{(2\pi)^4} \frac{e^{ipn} - 1}{\left(\sum_\mu 4 \sin^2 \frac{p_\mu}{2} \right)^2} \\ &\simeq \quad -\frac{1}{8\pi^2} \ln \left(|n| \right) , \quad (|n| >> 1). \end{aligned} \tag{3.47}$$

Note that the IR divergence associated to the propagator of π is actually canceled in this invariant correlation function. We assume this cancellation of the IR divergence holds at higher orders as in the two-dimensional nonlinear sigma model[20, 36]. Then $G(n)$ is a function of the dimensionless parameters $|x|/a \, (n = ax)$ and λ.

This two-point function should satisfy the renormalization group equation,

$$\left[-a \frac{\partial}{\partial a} + \beta(\lambda) \frac{\partial}{\partial \lambda} - 2\gamma_g(\lambda) \right] G_g(|x|/a, \lambda) = 0. \tag{3.48}$$

Using the leading order result Eq. (3.46), the anomalous dimension of g can be evaluated as

$$2\gamma_g = - \left(\frac{N^2 - 1}{2N} \right) \frac{1}{8\pi^2} \lambda^2 . \tag{3.49}$$

In order to see the long-distance behavior of the two-point function, we will solve the renormalization group equation and improve the two-point function by the renormalization group method. We first introduce the effective coupling constant λ_x at the distance $|x|$ by the equation

$$- x_\mu \frac{\partial}{\partial x_\mu} \lambda_x = \beta(\lambda_x), \quad \lambda_a = \lambda. \tag{3.50}$$

λ_a is identified with the bare coupling constant λ. Then the solution of the renormalization group equation can be written in the form

$$G_g(|x|/a, \lambda_a) = \exp\left(- \int_{\lambda_a}^{\lambda_x} d\lambda \frac{2\gamma_g(\lambda)}{\beta(\lambda)} \right) G_g(1, \lambda_x), \tag{3.51}$$

Using the leading order result Eq. (3.46) and the one-loop β and γ_g, we obtain

$$\begin{aligned} G_g(|x|/a, \lambda) \quad &= \quad \left(\frac{a}{|x|}\right)^{\left(\frac{N^2-1}{2N}\right)\frac{\lambda_x^2}{8\pi^2}} \times \\ & \qquad \left(1 + \lambda_x^2 \left(\frac{N^2-1}{2N}\right) [G(1) - G(0)]\right). \end{aligned} \tag{3.52}$$

$$\frac{1}{\lambda_x^2} = \frac{1}{\lambda_a^2} - \frac{N}{16\pi^2} \ln(|x|/a). \tag{3.53}$$

We should note that the validity of this improved two-point function is restricted in the region of the distance

$$a \leq |x| < \frac{1}{\Lambda_\alpha}, \tag{3.54}$$

where Λ_α stands for the renormalization group invariant mass scale of the pure gauge model:

$$\Lambda_\alpha = \frac{1}{a} \exp\left(- \frac{16\pi^2}{N} \frac{1}{\lambda^2} \right). \tag{3.55}$$

In spite of the restricted validity of the two-point function, its power behavior strongly suggests that the two-point function vanishes at large distance and therefore that *the global gauge symmetry does not break spontaneously at* $\beta = \infty$. Furthermore, the asymptotic freedom is suggesting the dynamical generation of the mass of the gauge freedom by the dimensional transmutation:

$$m = c\, \Lambda_\alpha. \tag{3.56}$$

From the point of view of the Foerster-Nielsen-Ninomiya mechanism[18], we see that the gauge symmetry breaking due to the covariant gauge fixing term and the Faddeev-Popov ghost action is very special; it gives the asymptotically free self-coupling of the gauge degree of freedom and it can be “large” without spoiling the disordered nature of the gauge freedom by virtue of the severe IR singularity which occurs when it is “large”. Moreover, the coupling could control the mass scale of the gauge freedom by the relation Eq. (3.56), if it would be generated dynamically. Note that the situation is quite similar to that of the two-dimensional nonabelian chiral gauge theory[19].

4 Perturbative Foerster-Nielsen-Ninomiya mechanism

4.1 Effect of explicit gauge symmetry breaking

The one-loop renormalizability and the symmetric realization of the global gauge symmetry due to the asymptotic freedom and the IR singularity are remarkable properties of the pure gauge model. However, since the model breaks gauge (BRST) invariance explicitly by the Wess-Zumino-Witten term, there is no reason based on symmetry to expect that the renormalizability also holds at higher orders. Rather, we expect the gauge non-invariant divergent and finite terms, which can be written in terms of the local operators of the dimension less than five, $\mathcal{O}_k$ $(k = 1, \cdots, 13)$, at some higher, but finite orders.

In this section, in order to examine the effect of such gauge breaking terms, we will consider the situation in which we have *the explicit gauge breaking term* corresponding to $\mathcal{O}_1 = -\sum_{n\mu} \frac{1}{2} A^a_{n\mu} A^a_{n\mu}$:

$$\begin{aligned} S_B &= K \sum_{n\mu} \mathrm{Tr}(U_{n\mu} + U^\dagger_{n\mu}) && (4.1) \\ &\longrightarrow K \sum_{n\mu} \mathrm{Tr}(g_n g^\dagger_{n+\mu} + g_{n+\mu} g^\dagger_n) \simeq -\sum_{n\mu} \frac{1}{2} K A^a_{n\mu} A^a_{n\mu}, && (4.2) \end{aligned}$$

and examine how this gauge symmetry breaking term would spoil the disordered nature of the gauge freedom and causes the system to fall into the broken phase, or how "small" such gauge symmetry breaking terms must be in order to keep the system in the symmetric phase. Note that this analysis is nothing but the analysis of the Foerster-Nielsen-Ninomiya mechanism[18] in the context of the pure gauge model. Here we can examine this dynamical issue *within the framework of the perturbation theory.*

From the point of view of the (continuum limit) perturbation theory of the nonabelian gauge theory, it is well-known that the introduction of the mass term of the vector boson does not affect the renormalizability of the theory, although it breaks BRST invariance explicitly. It is also known that based on this fact, the mass term can be used as an IR regulator for the massless vector boson[6]. We will see, as expected, that this is also true in the pure gauge model. Moreover, we will find a close relation between the prescription of the IR regularization by the mass term and the Foerster-Nielsen-Ninomiya mechanism.

4.2 Renormalizability with gauge symmetry breaking

Let us first check the renormalizability of the pure gauge model in the presence of the explicit gauge symmetry breaking term. We will repeat the one-loop

[6] A proof of the renormalizability of the nonabelian gauge theory with the mass term of the gauge field based on the Ward-Takahashi (Slavnov-Taylor) identity can be found, for example, in [37].

analysis by the background field method, including the gauge breaking term Eq. (4.2). It is expanded up to the quadratic terms in the quantum gauge fluctuation π as

$$\begin{aligned} S_B[\exp(i\lambda\pi)g] &= S_B[g] + S_{B\,0}[\pi^2] \\ &+ S_{B\,1}[\pi^2, \sin A] + S_{B\,2}[\pi^2, \cos A - 1] + \mathcal{O}(\pi^3). \end{aligned} \tag{4.3}$$

The explicit formula of these terms are given in the appendix F. The linear term is assumed to be included into the equation of motion. $S_{B\,0}[\pi^2]$, which is given as

$$S_{B\,0}[\pi^2] = -\frac{1}{2}K\lambda^2 \sum_{n\mu} \nabla_\mu \pi_n^a \nabla_\mu \pi_n^a, \tag{4.4}$$

gives a correction to the kinetic term of π. It modifies the propagator of π as follows:

$$\begin{aligned} \langle \pi_n^a \pi_m^b \rangle &= \delta^{ab} \int \frac{d^4p}{(2\pi)^4} e^{ip(n-m)} \frac{1}{\left(\sum_\mu 4\sin^2 \frac{p_\mu}{2}\right)^2 + M_0^2 \left(\sum_\mu 4\sin^2 \frac{p_\mu}{2}\right)} \\ &\equiv \delta^{ab} G_B(n-m). \end{aligned} \tag{4.5}$$

Here we have set

$$M_0^2 = K\lambda^2. \tag{4.6}$$

(We denote the dimensionless mass parameter with subscript "0" as M_0 and the dimensional one without it: $Ma = M_0$.) This correction does not change the ultraviolet(UV) behavior of the propagator, but does change the IR structure substantially: it is no more IR divergent in four-dimensions.

The quantum correction to the classical action are now divided into two classes: the first class consists of the contributions which does not have the gauge-breaking vertexes from S_B and are given by the same expressions as Eqs. (3.32), (3.33), (3.34), (3.35), (3.36) and (3.37). But these terms should be evaluated with the propagator given by Eq. (4.5) rather than Eq. (3.15). Taking account of the fact that M_0^2 should be counted to have mass dimension two, they are evaluated as

$$\begin{aligned} &\Delta S_1 + \Delta S_2 + \Delta S_{1c} + \Delta S_{2c} \\ &\quad = -\lambda^2 \left[N\bar{G}_B + \left(\frac{N^2-1}{N} [G_B(0) - G_B(1)] \right) \right] \mathcal{O}_0 \\ &\quad\quad + M_0^2 \left[N\tilde{G}_B - \left(\frac{N^2-1}{2N} \right) \nabla^2 G_B(0) \right] \mathcal{O}_1. \end{aligned} \tag{4.7}$$

where

$$\begin{aligned} \bar{G}_B &= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \frac{\left(\sum_\mu \sin^2 k_\mu\right)}{\left[\sum_\mu 4\sin^2 \frac{k_\mu}{2} + M_0^2\right]^3} \\ &\simeq -\frac{1}{16\pi^2} \ln(M_0) + \bar{C}_B \qquad (M_0 << 1), \end{aligned} \tag{4.8}$$

and

$$
\begin{aligned}
\tilde{G}_B &= \frac{3}{4}\int \frac{d^4k}{(2\pi)^4} \frac{\left(\sum_\mu \sin^2 k_\mu\right)}{\left[\left(\sum_\mu 4\sin^2\frac{k_\mu}{2}\right)^2\left(\sum_\mu 4\sin^2\frac{k_\mu}{2}+M_0^2\right)\right]} \\
&\simeq -\frac{3}{32\pi^2}\ln(M_0)+\tilde{C}_B \qquad (M_0 << 1). \qquad (4.9)
\end{aligned}
$$

The second class consists of the additional contribution due to the gauge-breaking vertexes. They are given as follows up to the second order in A_μ:

$$
\begin{aligned}
\Delta S_{B\,1}[A] &= \langle S_{B\,1}[A]\rangle_0, \qquad (4.10)\\
\Delta S_{B\,2}[A^2] &= \langle S_{B\,2}[A^2]\rangle_0 + \frac{1}{2}\langle S_{B\,1}[A]^2\rangle_0 + \langle S_{B\,1}[A]S_1[A]\rangle_0. \qquad (4.11)
\end{aligned}
$$

They are evaluated as follows:

$$
\Delta S_{B\,1}+\Delta S_{B\,2} = -M_0^2\left(\frac{N^2-1}{2N}\right)[G_B(0)-G_B(1)]\ \mathcal{O}_1. \qquad (4.12)
$$

From these results, we can see that the pure gauge model with the explicit gauge symmetry breaking Eq. (4.2) is actually renormalizable at one-loop. The renormalization of λ is same as before and it is asymptotically free. $M_0^2 = K\lambda^2$ is multiplitically renormalized.

$$
\begin{aligned}
\frac{1}{\lambda^2}+\frac{N}{16\pi^2}\ln(a\mu) &= \frac{1}{\lambda_R^2}, \qquad (4.13)\\
\frac{M^2}{\lambda^2}\left(1-\lambda^2\frac{5N}{32\pi^2}\ln(a\mu)\right) &= \frac{M_R^2}{\lambda_R^2}. \qquad (4.14)
\end{aligned}
$$

μ should be understood as a certain renormalization point.

4.3 Restoration of chiral $SU(N)$ global symmetry

Let us next examine the realization of the chiral $SU(N)$ global symmetry. Since the IR singularity of the pure gauge model is now "regularized" by the introduction of the explicit gauge symmetry breaking term, we may examine the local order parameter

$$
\left\langle g_{n\,i}^{\ j}\right\rangle. \qquad (4.15)
$$

At the leading non-trivial order in the perturbation expansion, it is evaluated as

$$
\left\langle g_{n\,i}^{\ j}\right\rangle = \delta_i^j\left(1-\frac{\lambda^2}{2!}\left(\frac{N^2-1}{2N}\right)G_B(0)\right).
$$

For a generic K and a small λ, it does not vanish and the chiral $SU(N)$ symmetry breaks down to its vector subgroup $SU(N)_V$.

However, the second term is potentially IR(UV) divergent and it becomes large as K decreases:

$$G_B(0) \quad = \quad -\frac{1}{16\pi^2}\ln K\lambda^2 + C_B \qquad (0 < K << 1). \tag{4.16}$$

It could be large so that it cancels the first term. As a crude approximation, the critical point at which the local order parameter vanishes can be estimated from this equation as

$$K_c\lambda^2 \simeq \exp\left\{-32\pi^2\left(\frac{2N}{N^2-1}\right)\left[\frac{1}{\lambda^2}-\left(\frac{N^2-1}{4N}\right)C_B\right]\right\}. \tag{4.17}$$

From this relation, we can see that, if K is sufficiently small so that $0 \le K < K_c$, the chiral $SU(N)$ global symmetry can be restored.

We may examine the invariant two-point function as before. At the leading order, it reads

$$G_{gB}(n) = 1 + \lambda^2\left(\frac{N^2-1}{2N}\right)\left[G_B(n) - G_B(0)\right]. \tag{4.18}$$

From the long distance behavior of $G_B(n)$ for $|n| >> 1$, we should be able to extract the realization of the chiral $SU(N)$. But in this case, since we have introduced the parameter $aM = M_0$ which has mass dimension, the long distance limit can depend on the relative scale to this parameter.

In fact, the long distance behavior which corresponds to the spontaneous breakdown of the chiral $SU(N)$ appears in the limit:

$$|n| \;>>\; \frac{1}{M_0} \;\simeq\; 1. \tag{4.19}$$

In this limit, we can see that $G_B(n)$ falls off in power and exponentially for large $|n|$, by decomposing it into the propagators of the massless scalar and the massive ghost:

$$\begin{aligned} G_B(n) \quad &= \quad \int \frac{d^4p}{(2\pi)^4}\, e^{ipn}\, \frac{1}{M_0^2}\left(\frac{1}{\sum_\mu 4\sin^2\frac{p_\mu}{2}} - \frac{1}{\sum_\mu 4\sin^2\frac{p_\mu}{2}+M_0^2}\right) \\ &\simeq \quad A\frac{1}{M_0^2|n|^2} - Be^{-cM_0|n|} \qquad (M_0|n| > 1). \end{aligned} \tag{4.20}$$

Then the invariant two-point function does not vanish at large distance:

$$G_B(n) \longrightarrow \frac{1}{N}\mathrm{Tr}\left\{\langle g\rangle\langle g\rangle\right\} \neq 0, \quad (|n| \longrightarrow \infty), \tag{4.21}$$

indicating the spontaneous breakdown of the chiral $SU(N)$ global symmetry. Thus the "large" gauge symmetry breaking term causes the system to fall into the broken phase. Here "large" means that

$$M_0 = K\lambda^2 \simeq \mathcal{O}(1), \qquad \text{or} \qquad M \simeq \mathcal{O}\left(\frac{1}{a}\right). \tag{4.22}$$

On the other hand, we may consider another long distance limit such that

$$\frac{1}{M_0} > |n| \;>>\; 1. \tag{4.23}$$

Then the behavior of $G_B(n)$ is logarithmic,

$$G_B(n) \quad \simeq \quad -\frac{1}{8\pi^2}\ln\left(M_0|n|\right), \qquad (0 < M_0|n| < 1). \tag{4.24}$$

This means that the invariant two-point function decreases at the long distance by the power law. Since the perturbative calculation is subject to the restriction of the validity for the long distance due to the asymptotic freedom, this second limit is a possible alternative as long as M_0 is "small" so that

$$1 \;<<|n| \;<\; \exp\left(\frac{16\pi^2}{N}\frac{1}{\lambda^2}\right) \;<\; \frac{1}{M_0}. \tag{4.25}$$

This relation is giving the criterion about how "small" the gauge symmetry breaking term M_0 must be for the chiral $SU(N)$ global symmetry to be restored at long distance. Note that this condition is consistent with the previous crude estimation of the critical point.

Based on the renormalizability, the two-point function can be examined by the renormalization group equation, which in this case reads

$$\left[-a\frac{\partial}{\partial a} + \beta(\lambda)\frac{\partial}{\partial\lambda} - \gamma_M(\lambda)M^2\frac{\partial}{\partial M^2} - 2\gamma_g(\lambda)\right]G_g(|x|/a, \lambda, aM) = 0. \tag{4.26}$$

Here we assume the mass independent renormalization scheme[38] for a technical convenience. From Eq. (4.14), we obtain the anomalous dimension of M^2 as

$$\gamma_M(\lambda) = a\frac{\partial \ln M^2}{\partial a} = \frac{5N}{64\pi^2}\lambda^2. \tag{4.27}$$

Using the leading order result Eq. (4.18), we also obtain the anomalous dimension of g as before

$$2\gamma_g = -\left(\frac{N^2-1}{2N}\right)\frac{1}{8\pi^2}\lambda^2. \tag{4.28}$$

We introduce the effective coupling constant λ_x and the effective mass M_x^2 at the distance $|x|$ by the equation

$$-\,x_\mu\frac{\partial}{\partial x_\mu}\lambda_x \quad = \quad \beta(\lambda_x); \quad \lambda_a = \lambda, \tag{4.29}$$

$$x_\mu\frac{\partial}{\partial x_\mu}\ln M_x^2 \quad = \quad \gamma_M(\lambda_x); \quad M_a^2 = M^2. \tag{4.30}$$

λ_a and M_a^2 are identified with the bare coupling constants λ and M^2. Then the solution of the renormalization group equation can be written in the form

$$G_g(|x|/a, \lambda, aM) = \exp\left(-\int_{\lambda_a}^{\lambda_x} d\lambda\frac{2\gamma_g(\lambda)}{\beta(\lambda)}\right)G_g(1, \lambda_x, aM_x), \tag{4.31}$$

Using the leading order result Eq. (3.46), and the one-loop β, γ_g and γ_M, we obtain

$$G_g(|x|/a, \lambda, aM) = \left(\frac{a}{|x|}\right)^{\left(\frac{N^2-1}{2N}\right)\frac{\lambda_x^2}{8\pi^2}} \times \left(1 + \lambda_x^2 \left(\frac{N^2-1}{2N}\right) [G_B(1) - G_B(0)] (aM_x)\right). \tag{4.32}$$

$$M_x^2 = M_a^2 \left(\frac{|x|}{a}\right)^{\frac{5N}{64\pi^2}\lambda_x^2}. \tag{4.33}$$

From this expression, we can see that the invariant two-point function has the power behavior same as before, as long as the following relation among the scales is satisfied:

$$a << |x| < \frac{1}{\Lambda_\alpha} < \frac{1}{M_{\Lambda_\alpha}}. \tag{4.34}$$

This is the renormalized version of the criterion for the restoration of the chiral $SU(N)$ global symmetry. From the point of view of the prescription of the IR regularization, this condition is just telling that the IR regularization mass M must be smaller than the scale of the theory Λ_α so that its effect should be irrelevant physically.

4.4 Phase diagram of the pure gauge model

This criterion for the restoration of the chiral $SU(N)$ global symmetry can be regarded to define the phase boundary between the symmetric phase and the broken phase in the coupling space $(\frac{1}{\alpha}, K)$ for the region of the small λ and K:

$$K_c(\lambda) \simeq \frac{1}{\lambda^2} \exp\left(-\frac{16\pi^2}{N}\frac{1}{\lambda^2}\right). \tag{4.35}$$

We can see that, although it is tiny for a small λ, a finite region of K can exist, where the chiral $SU(N)$ global symmetry is restored. Namely, if K is sufficiently small so that $0 \leq K < K_c(\lambda)$, the chiral $SU(N)$ would not be broken spontaneously. As λ becomes large, this symmetric region is expected to become larger, because the bound becomes milder.

If no phase transition would occur in the course of removing the covariant gauge fixing term along the limit:

$$\lambda \to \infty, \qquad K = 0, \tag{4.36}$$

then it is naturally expected that the critical line $K_c(\lambda)$ finally reaches the critical point of the usual four-dimensional $SU(N)$ non-linear sigma model (coupled to the fermion and ghost fields), $K_c(\infty)$. In the case of the anomaly-free theory, this assumption seems plausible. Then the expected phase diagram in the

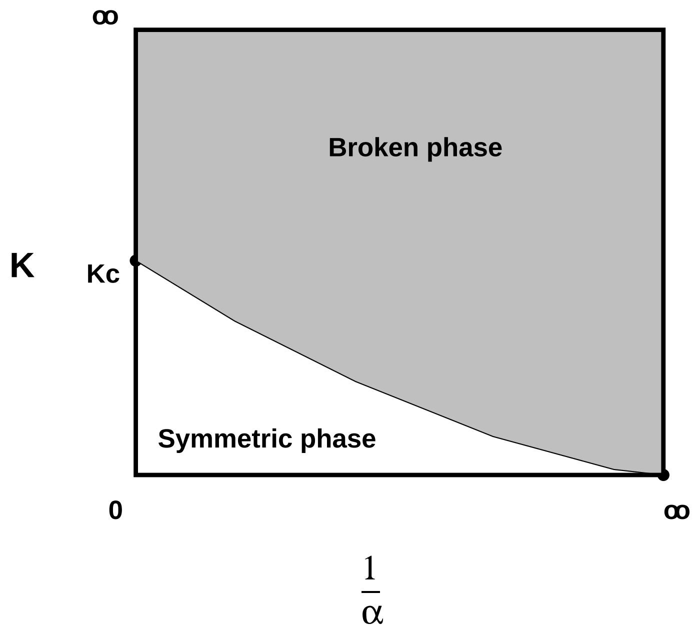

Figure 1: Possible phase structure for the pure gauge model of anomaly-free $SU(N)$ non-abelian chiral gauge theory.

coupling space of $(1/\alpha, K)$ is given by Fig. 1. We should note that the existence of the symmetric phase in the region of the small λ can be possible here due to the IR singularity and the non-abelian nature of the pure gauge dynamics, namely the asymptotic freedom.

By the mean-field method, the phase structure has been examined in [24] for the similar model with the higher derivative coupling term which is induced from the covariant-type gauge fixing action proposed in [25]. According to this result, the paramagnetic (PM) phase terminates at a finite value of $1/\alpha$ on the line $K = 0$. Beyond this point, the critical line which separates the ferromagnetic phase(FM) and the helicoidal-ferromagnetic phase(FMD) lies along the $K = 0$ line to the limit $1/\alpha = \infty$.

The point we make here is the possibility that this transition point among PM, FM and FMD phases and the phase boundary line of FM and FMD phases can be pushed away to the limit $1/\alpha = \infty$, due to the non-abelian nature of the gauge freedom. By virtue of the asymptotic freedom, the mass of the gauge freedom can be generated and the paramagnetic(PM) phase can persist up to the UV fixed point of α at $1/\alpha = \infty$. Our analysis of the effect of the gauge symmetry breaking term based on the perturbation theory is suggesting this possibility, supporting the picture of the dynamical restoration of the chiral $SU(N)$ symmetry given by Hata[20]. This dynamical restoration is protected from the "small" gauge symmetry breaking by the Foerster-Nielsen-Ninomiya

mechanism.

On the other hand, if an IR fixed point of λ could occur, the disordered phase which is found in the perturbation theory, may not necessarily be connected to the disordered phase of the usual four-dimensional non-linear sigma model along the line

$$0 \leq K \leq K_c(\infty), \qquad \lambda = \infty. \tag{4.37}$$

In the case of the two-dimensional nonabelian chiral gauge theory, the quantum effect of anomaly (Wess-Zumino-Witten term) produces such an IR fixed point. It is quite interesting if it could happen in four-dimensions, because it would be possible to distinguish the anomalous theory from the anomaly-free theory by the dynamical nature of the gauge degree of freedom at $\beta = \infty$. To explore this possibility, we need to examine the effects of the higher order quantum corrections in both anomalous and anomaly-free theories, which is beyond the scope of this article.

In order to examine the phase structure of the pure gauge model fully, we need the analysis beyond the perturbation theory. We will leave this issue for future study. The lattice formulation of the four-dimensional pure gauge model and the possibility to examine it by the Monte Carlo simulation will be discussed elsewhere[21].

4.5 Induced gauge symmetry breaking and counter term

Next we consider the effect of the induced gauge symmetry breaking terms from the Wess-Zumino-Witten term. Since there is no reason based on the symmetry, we cannot escape them. Here we consider the breaking term of dimension two, $\mathcal{O}_1$, as before. Let us assume the situation in which we get additive contributions of the induced gauge breaking in the coupling K and it is written effectively in the form

$$K_{\text{eff}} = K + K_{\text{ind}}(\lambda). \tag{4.38}$$

By the consideration based on the charge conjugation and the anomaly cancellation, the induced term is expected to appear first at the three-loop order[7]:

$$K_{\text{ind}}(\lambda) = \sum_{l \geq 3} \Delta_l \lambda^{2l}. \tag{4.39}$$

Since $K_c(\lambda)$ has the essential singularity in λ, any polynomial of λ cannot satisfy the criterion of the symmetry restoration for small λ. This means that any perturbatively induced term can cause the spontaneous break down of the chiral $SU(N)$. Therefore we are forced to introduce the counter term as a part of the explicit breaking term so that K_{eff} satisfies

$$0 < K_{\text{eff}} < K_c(\lambda), \tag{4.40}$$

where

$$K_{\text{eff}} = K + K_{\text{ind}}(\lambda) + \delta K_{\text{count}}. \tag{4.41}$$

[7]The author owes this point to the discussion with A. Yamada.

This fine tuning can be achieved, since the finite region of K for the symmetry restoration can exist for the nonabelian gauge theory, as we have argued in the previous subsection. As λ becomes larger, we can expect that this fine tuning becomes milder. With this counter terms, the IR structure of the pure gauge model can be maintained and all the properties of the pure gauge model hold true.

5 Disordered gauge freedom and chiral fermion

Based on the dynamical features observed in the previous section, we assume that the global gauge symmetry does not break spontaneously and the gauge freedom acquires mass dynamically in the pure gauge limit for the entire region of the gauge parameter. Under this assumption, we examine the spectrum of the fermion correlation functions in the vacuum overlap formulation. Our strategy is, with the help of the asymptotic freedom, to tame the gauge fluctuation by approaching the critical point of the gauge degree of freedom without spoiling its disordered nature. Namely, we invoke the perturbation (spinwave) expansion in order to examine the fermion spectrum in *the disordered phase*.

5.1 Overlap correlation function

Before discussing the dynamical effect of the disordered gauge freedom to the fermion spectrum in lattice chiral gauge theory, let us first summarize what happens in the pure gauge limit of the lattice QCD. In the limit of vanishing gauge coupling, the gauge variable is restricted to the pure gauge form

$$U_{n\mu} = g_n g_{n+\mu}^\dagger, \quad g_n \in SU(3) \qquad (\beta = \infty). \tag{5.1}$$

Then the quark action reads

$$\sum_n \left\{ \frac{1}{2} \sum_\mu \left(\bar{q}_n (1-\gamma_\mu) g_n g_{n+\hat{\mu}}^\dagger q_{n+\hat{\mu}} + \bar{q}_{n+\hat{\mu}} (1+\gamma_\mu) g_{n+\hat{\mu}} g_n^\dagger q_n \right) - m_0\, \bar{q}_n q_n \right\}. \tag{5.2}$$

Since the action still possess the local gauge invariance, we cannot identify directly the "colored" quark even in the pure gauge limit through the gauge non-invariant correlation function of the quark:

$$\left\langle q_{ni} \bar{q}_m{}^j \right\rangle. \tag{5.3}$$

This is because it must vanish according to the Elitzur's theorem[39].

On the other hand, by the change of variable,

$$q_{ni} \longrightarrow \psi_{ni} \equiv \left(g_n^\dagger\right)_i^j q_{nj}, \tag{5.4}$$

the action turns into that of the free Wilson fermion. The functional integral measure of the quark field is invariant under the change of variable. Therefore

the gauge freedom decouples completely from the fermion. This action has a symmetry under the global transformation,

$$\psi_{ni} \rightarrow h_i^j \psi_{nj} \quad h \in SU(3), \tag{5.5}$$

This is not the global gauge symmetry but comes from the arbitrariness of choice of the pure gauge variable. We refer this as $SU(3)_H$. ψ_n belongs to the fundamental representation of $SU(3)_H$.

The fermion spectrum is measurable by the gauge invariant correlation function

$$\langle \psi_{ni} \bar{\psi}_m{}^j \rangle, \tag{5.6}$$

which is given by the correlation function of the free Wilson fermion. By virtue of the Wilson term, the masses of the species doubles are lifted to cutoff scale. Since ψ_n belongs to the fundamental representation of the $SU(3)_H$, the correct number of light *Dirac* fermions emerge just same as the quark in the fundamental representation of $SU(3)_{Color}$.

In the lattice chiral gauge theory based on the vacuum overlap, the correlation functions of fermion are defined by putting creation and annihilation operators in the overlap of vacua with different masses. We refer the correlation function in this definition as *overlap correlation function*. The two-point overlap correlation function of the fermion in the representation r is defined as follows:

$$\begin{aligned} \langle a_{ni} a_m^{\dagger\, j} \rangle_r \quad \equiv \quad & \frac{1}{Z} \int [dU][d\bar{c}dc] \exp \left(-\beta S_G - \frac{1}{2\alpha} \left(\bar{\nabla} \hat{A} \right)^2 - \bar{c} M[U] c \right) \times \\ & \prod_{rep.} \left(\frac{\langle + | v+ \rangle}{|\langle + | v+ \rangle|} \langle v+ | v- \rangle \frac{\langle v- | - \rangle}{|\langle v- | - \rangle|} \right) \times \\ & \langle v+ | \left\{ \hat{a}_{ni} \hat{a}_m^{\dagger\, j} - \frac{1}{2} \delta_{nm} \delta_i^j \right\} | v- \rangle_r \Big/ \langle v+ | v- \rangle_r . \end{aligned} \tag{5.7}$$

This correlation function does not transform covariantly under the local gauge transformation because of the explicit gauge symmetry breaking. Only under the global gauge transformation Eq. (2.26), it does.

In the pure gauge limit, it turns out to be

$$\begin{aligned} \langle a_{ni} a_m^{\dagger\, j} \rangle_r \quad = \quad & \frac{1}{Z} \int d\mu[g; \alpha] \times \\ & \langle + | \hat{G}^\dagger \left\{ \hat{a}_{ni} \hat{a}_m^{\dagger\, j} - \frac{1}{2} \delta_{nm} \delta_i^j \right\} \hat{G} | - \rangle_r \Big/ \langle + | - \rangle_r . \end{aligned} \tag{5.8}$$

By noting the transformation property of the creation and annihilation operators under the action of $\hat{G}$, we can see that this correlation function has the structure of the product of the free fermion two-point correlator and the correlation function of the gauge degree of freedom:

$$\left\langle \left(g_{n\,i}^{\ k} \right)_r \left(g^{\dagger\, j}_{ml} \right)_r \right\rangle \times \langle + | \left\{ \hat{a}_{nk} \hat{a}_m^{\dagger\, l} - \frac{1}{2} \delta_{nm} \delta_k^l \right\} | - \rangle_r \Big/ \langle + | - \rangle_r . \tag{5.9}$$

If the gauge degree of freedom would be disordered and would have a very short correlation length of the order of the lattice spacing, it is hardly expected that we could find a physical fermion in this correlation function, even when the free fermion two-point correlator could have a pole of physical particle. In fact, according to our perturbative analysis with the covariant gauge fixing term given in the previous section, the non-invariant correlation functions suffer from the IR divergence in general and they cannot be regarded as the physical observables. We may think of the invariant counterpart of the above correlation function by taking the trace over the group indexes. However, since the invariant correlation function of the gauge degree of freedom shows the power behavior

$$\left\langle \frac{1}{N} \mathrm{Tr} \left\{ g_n g_0^\dagger \right\} \right\rangle = \left(\frac{1}{|n|} \right)^{\left(\frac{N^2-1}{2N} \right) \frac{\lambda^2}{8\pi^2}}, \tag{5.10}$$

its combolution with the free fermion correlator cannot describe a physical free particle[40]. We may regard this screening phenomenon as the counterpart of the Elitzur's theorem in the Lattice QCD. The theorem, as noticed in [16], does not seem possible to be established in the lattice chiral gauge theory defined by the vacuum overlap because of the nonlocal nature of the fermion determinant, even when it is formulated with the manifest local gauge invariance by the explicit integration over the gauge degree of freedom. But, due to the disordered gauge degree of freedom, the screening of the "color charge" can take place effectively. This correspondence is quite analogous to the mapping of the dynamical features between the two-dimensional nonlinear sigma model and the four-dimensional Yang-Mills theory.

On the other hand, just in the same manner as the lattice QCD at $\beta = \infty$ as we reviewed above, we can consider the change of variables

$$a_{ni} \longrightarrow b_{ni} = g^{\dagger\, j}_{ni} a_{nj}. \tag{5.11}$$

And we may define another fermion two-point correlation function by

$$\begin{aligned} \langle b_{ni} b^{\dagger\, j}_{m} \rangle_r \quad &\equiv \quad \frac{1}{Z} \int d\mu[g;\alpha] \times \\ &\qquad \langle + | \hat{G}^\dagger \left\{ \hat{b}_{ni} \hat{b}^{\dagger\, j}_{m} - \frac{1}{2} \delta_{nm} \delta_i^j \right\} \hat{G} | - \rangle_r \Big/ \langle + | - \rangle_r . \\ &= \quad \langle + | \left\{ \hat{a}_{nk} \hat{a}^{\dagger\, l}_{m} - \frac{1}{2} \delta_{nm} \delta_k^l \right\} | - \rangle_r \Big/ \langle + | - \rangle_r . \end{aligned} \tag{5.12}$$

This correlation function is invariant under the global gauge transformation $SU(N)_C$. It transforms covariantly under the global transformation $SU(N)_H$ given by Eq. (2.27). It was shown in [16] that free *Weyl* fermions appear from this correlation function, which degeneracy is just same as that of the continuum *Weyl* fermion in the representation r. The appearance of poles of species doublers are suppressed by the appearance of zeros at the vanishing momenta

for doublers.[8] Note also that the gauge degree of freedom completely decouples from this correlation function. This is a remarkable property of the overlap correlation function. It makes it easy and clear to identify the fermion spectrum in the pure gauge limit.

By these observations, we can see that the way the Weyl fermion emerges in the pure gauge limit in the overlap correlation function is quite analogous to the way the Dirac fermion does in the pure gauge limit of the lattice QCD, although the mechanism to suppress the species doublers is quite different.

5.2 Boundary fermion correlation function

However, through the analysis of the waveguide model[11], it has been claimed that the required disordered nature of the gauge freedom causes the vector-like spectrum of fermion[12]. In this argument, the fermion correlation functions at the waveguide boundaries were examined. One may think of the counter parts of these correlation functions in the overlap formulation by putting creation and annihilation operators in the overlap of vacua with the same signature of mass. Let us refer this kind of correlation function as *boundary correlation function*. We should note that the boundary correlation functions are no more the observables in the sense defined in the overlap formulation[16]; they cannot be expressed by the overlap of two vacua with their phases fixed by the Wigner-Brillouin phase convention. But, they are still relevant because they can probe the auxiliary fermionic system for the definition of the complex phase of chiral determinant and therefore the anomaly (the Wess-Zumino-Witten term). If massless chiral states could actually appear in the boundary correlation functions, we would have difficulty defining the complex phase.

As for the overlap of the vacuum of the Hamiltonian with negative mass, there are three possible definitions of boundary correlation function in the representation r:

$$\langle \phi_{ni} \phi^{\dagger}_{m}{}^{j} \rangle_{-r} \equiv \frac{1}{Z} \int d\mu[g;\alpha] \frac{\langle - | \hat{G}^{\dagger} \left\{ \hat{a}_{ni} \hat{a}^{\dagger}_{m}{}^{j} - \frac{1}{2} \delta_{nm} \delta_i^j \right\} | - \rangle_r}{\langle - | \hat{G}^{\dagger} | - \rangle_r}, \tag{5.13}$$

$$\langle \varphi_{ni} \varphi^{\dagger}_{m}{}^{j} \rangle_{-r} \equiv \frac{1}{Z} \int d\mu[g;\alpha] \frac{\langle - | \left\{ \hat{a}_{ni} \hat{a}^{\dagger}_{m}{}^{j} - \frac{1}{2} \delta_{nm} \delta_i^j \right\} \hat{G}^{\dagger} | - \rangle_r}{\langle - | \hat{G}^{\dagger} | - \rangle_r}, \tag{5.14}$$

$$\langle \varphi_{ni} \phi^{\dagger}_{m}{}^{j} \rangle_{-r} \equiv \frac{1}{Z} \int d\mu[g;\alpha] \frac{\langle - | \left\{ \hat{a}_{ni} \hat{G}^{\dagger} \hat{a}^{\dagger}_{m}{}^{j} - \frac{1}{2} \delta_{nm} \left(g^{\dagger}_{m} \right)_i^j \right\} | - \rangle_r}{\langle - | \hat{G}^{\dagger} | - \rangle_r}. \tag{5.15}$$

[8]This has been pointed out to be a possible way out of the Nielsen-Ninomiya theorem.[41, 42] However, the examples are known in which they cause ghost states which contribute to the vacuum polarization with wrong signature and lead to the wrong normalization[43, 44]. In the case of the vacuum overlap formulation, it has been shown that the perturbative calculation gives the correct normalization of the vacuum polarization[45, 34, 46]. This result is naturally understood from the point of view of the infinite number of the Pauli-Villars fields[47]. Still it is desirable to clarify this point in relation to the Ward identity[44].

The transformation properties of these correlation functions under the chiral $SU(N)$ can be read as follows:

$$\langle \phi_{ni} \phi^{\dagger\ j}_{m} \rangle_{-r} \longrightarrow (g_0{}^s_i) \langle \phi_{ns} \phi^{\dagger\ t}_{m} \rangle_{-r} (g^{\dagger j}_{0t}), \tag{5.16}$$

$$\langle \varphi_{ni} \varphi^{\dagger\ j}_{m} \rangle_{-r} \longrightarrow (h^s_i) \langle \varphi_{ns} \varphi^{\dagger\ t}_{m} \rangle_{-r} (h^{\dagger j}_{\ t}), \tag{5.17}$$

$$\langle \varphi_{ni} \phi^{\dagger\ j}_{m} \rangle_{-r} \longrightarrow (h^s_i) \langle \varphi_{ns} \phi^{\dagger\ t}_{m} \rangle_{-r} (g^{\dagger j}_{0t}). \tag{5.18}$$

Note that contrary to the case of the overlap correlation functions, the gauge freedom never decouple from these boundary correlation functions. Then, due to the severe IR singularity, only chiral $SU(N)$ invariant quantities can be used as observables[36]. The first and second correlation functions can be made invariant under the chiral $SU(N)$ by taking the trace over the group indices. The third one cannot be made invariant and we should discard it from observables.

We will examine these two invariant correlation functions associated to the overlap of the vacuum of the Hamiltonian with negative mass in the following. As for the positive mass case, there are similarly three possible definitions of boundary correlation function in the representation r. They could be examined in a similar manner as the boundary correlation function associated with the negative mass.

5.2.1 Expression of boundary correlation functions

The invariant boundary correlation functions associated to the overlap of the vacuum of the Hamiltonian with negative mass are evaluated as follows:

$$\begin{aligned}
\langle \phi_{ni} \phi^{\dagger\ i}_{m} \rangle_{-r} &\equiv \frac{1}{Z} \int d\mu[g;\alpha] \frac{\langle - | \hat{G}^\dagger \left\{ \hat{a}_{ni} \hat{a}^{\dagger\ i}_{m} - \frac{1}{2}\delta_{nm}\delta^i_i \right\} | - \rangle_r}{\langle - | \hat{G}^\dagger | - \rangle_r} \\
&= \frac{1}{Z} \int d\mu[g;\alpha] \left[\frac{1}{2}\delta_{nm}\delta^i_i - S^v_-[g](n;m)^o_i \left(g^{\dagger\ i}_{mo} \right) \right], \qquad (5.19) \\
\langle \varphi_{ni} \varphi^{\dagger\ i}_{m} \rangle_{-r} &\equiv \frac{1}{Z} \int d\mu[g;\alpha] \frac{\langle - | \left\{ \hat{a}_{ni} \hat{a}^{\dagger\ i}_{m} - \frac{1}{2}\delta_{nm}\delta^i_i \right\} \hat{G}^\dagger | - \rangle_r}{\langle - | \hat{G}^\dagger | - \rangle_r} \\
&= \frac{1}{Z} \int d\mu[g;\alpha] \left[\frac{1}{2}\delta_{nm}\delta^i_i - \left(g^{\dagger\ o}_{mi} \right) S^v_-[g](n;m)^i_o \right], \qquad (5.20)
\end{aligned}$$

where

$$\begin{aligned}
S^v_-[g](n,m)^j_i \equiv & \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \times \\
& e^{ipn} v_-(p) \left[v^\dagger_-(q) e^{-iqr} \left(g^{\dagger i}_{rj} \right) e^{ipr} v_-(p) \right]^{-1}_{(p,i)(q,j)} v^\dagger_-(q) e^{-iqm}.
\end{aligned} \tag{5.21}$$

We refer the reader to [19] for the detail of the calculation.

In the perturbation theory in λ, $S^v_+[g](n,m)$ can be expanded as follows:

$$\begin{aligned}
S^v_-[g](n,m)^o_s &= S^v_-(n-m)\,(\mathbb{1}^o_s) - \sum_r S^v_-(n-r)\,((-i\lambda)\pi_{r\,s}^{\ o})\,S^v_-(r-m) \\
&\quad -\sum_r S^v_-(n-r)\left(\frac{(-i\lambda)^2}{2!}\pi^{2\,o}_{r\,s}\right)S^v_-(r-m) \\
&\quad +\sum_{r,l} S^v_-(n-r)\,((-i\lambda)\pi_{r\,s}^{\ u})\,S^v_-(r-l)\,((-i\lambda)\pi_{l\,u}^{\ o})\,S^v_-(l-m) \\
&\quad +\mathcal{O}(\lambda^3).
\end{aligned} \tag{5.22}$$

Then we obtain at the one-loop order

$$\begin{aligned}
\langle\phi_{n\,i}\phi^{\dagger\ i}_m\rangle_{-r} &= \frac{1}{2}\delta_{nm}\delta^i_i - S^v_-(n-m)\delta^i_i \\
&- \lambda^2\sum_r S^v_-(n-r)\left[\langle\pi_{r\,i}^{\ o}\pi_{m\,o}^{\ i}\rangle' S^v_-(r-m)\right] \\
&+ \lambda^2\sum_{r,l} S^v_-(n-r)\left[\langle\pi_{r\,i}^{\ o}\pi_{l\,o}^{\ i}\rangle' S^v_-(r-l)\right]S^v_-(l-m) + \mathcal{O}(\lambda^4),
\end{aligned} \tag{5.23}$$

$$\begin{aligned}
\langle\varphi_{n\,i}\varphi^{\dagger\ i}_m\rangle_{-r} &= \frac{1}{2}\delta_{nm}\delta^i_i - S^v_-(n-m)\delta^i_i \\
&- \lambda^2\sum_r \left[\langle\pi_{n\,i}^{\ o}\pi_{r\,o}^{\ i}\rangle' S^v_-(n-r)\right]S^v_-(r-m) \\
&+ \lambda^2\sum_{r,l} S^v_-(n-r)\left[\langle\pi_{r\,i}^{\ o}\pi_{l\,o}^{\ i}\rangle' S^v_-(r-l)\right]S^v_-(l-m) + \mathcal{O}(\lambda^4),
\end{aligned} \tag{5.24}$$

$$\langle\pi_{r\,i}^{\ o}\pi_{l\,o}^{\ i}\rangle' = \delta^i_i\int\frac{d^4p}{(2\pi)^4}\frac{e^{ip(r-l)}-1}{\left(\sum_\mu 4\sin^2\frac{p_\mu}{2}\right)^2}. \tag{5.25}$$

Note that the IR divergences associated to the correlation function of the gauge freedom

$$\langle\pi_{r\,i}^{\ o}\pi_{l\,o}^{\ i}\rangle = \delta^i_i\int\frac{d^4p}{(2\pi)^2}\frac{e^{ip(r-l)}}{\left(\sum_\mu 4\sin^2\frac{p_\mu}{2}\right)^2}, \tag{5.26}$$

cancel among the second and third terms in the r.h.s. of Eqs. (5.23) and (5.24). To show this fact explicitly, we have used $\langle\pi_{r\,i}^{\ o}\pi_{l\,o}^{\ i}\rangle'$ instead.[9]

5.2.2 Boundary correlation functions at criticality

At the critical point, $\lambda=0$, the correlation functions reduce to the expression:

$$\langle\phi_{n\,i}\phi^{\dagger\ i}_m\rangle_{-r} = \langle\varphi_{n\,i}\varphi^{\dagger\ i}_m\rangle_{-r}, \tag{5.27}$$

[9] It is not difficult to see that the infrared divergence remains in the correlation function of the type of Eq. (5.15).

$$
\begin{aligned}
\langle \varphi_{ni} \varphi_m^{\dagger\, i} \rangle_{-r} &= \int \frac{d^4 p}{(2\pi)^4} e^{ip(n-m)} \delta_i^i \times \\
& \frac{1}{2\lambda_-} \begin{pmatrix} -m_0 + B(p) & C(p) \\ C^\dagger(p) & m_0 - B(p) \end{pmatrix}, \qquad (5.28)
\end{aligned}
$$

where

$$
\begin{aligned}
\lambda_- &= \sqrt{C_\mu^2(p) + (B(p) - m_0)^2}, && (5.29)\\
C(p) &= i\sigma_\mu \sin p_\mu = i\sigma_\mu C_\mu(p), && (5.30)\\
B(p) &= \sum_\mu (1 - \cos p_\mu). && (5.31)
\end{aligned}
$$

This boundary correlation function does not show any pole which can be interpreted as particle. Rather, it consists of the continuum spectrum with a mass gap. To see it, we consider the boundary correlation function without the spinor structure for simplicity. For the fixed spatial momentum p_k $(k = 1, 2, 3)$, the correlation function can be evaluated as

$$
\begin{aligned}
D(n_4; p_k) &= \int \frac{dp_4}{(2\pi)} e^{ip_4 n_4} \frac{1}{2\lambda_-} \\
&= \int \frac{dp_4}{(2\pi)} \frac{d\omega}{(2\pi)} e^{ip_4 n_4} \frac{1}{\omega^2 + \lambda_-^2} \\
&= \int \frac{d\omega}{(2\pi)} \frac{1}{(1 - \sum_k (1 - \cos p_k) - m_0)} \frac{e^{-M(\omega, p_k)|n_4|}}{2 \sinh M(\omega, p_k)},
\end{aligned}
$$

where

$$
\cosh M(\omega, p_k) = 1 + \frac{\omega^2 + \sum_k \sin^2(p_k) + (\sum_k (1 - \cos p_k) - m_0)^2}{2\,(1 - \sum_k (1 - \cos p_k) - m_0)}. \qquad (5.32)
$$

The minimum of $M(\omega, p_k)$ can be identified as the mass gap. For $m_0 = 0.5$, it appears at $\omega = 0$ and $p_k = 0$. In this case, the mass gap M_B is given by

$$
\cosh M_B = 1 + \frac{m_0^2}{2(1 - m_0)}. \qquad (5.33)
$$

Since the mass gap is of order of the cutoff, no light physical particle emerges in the boundary correlation functions on the critical point. They have very short-distance nature.[10]

5.2.3 Boundary correlation functions off criticality

Once we know that the boundary correlation functions have short distance nature at $\lambda = 0$, the quantum correction to such quantities can be evaluated by the perturbation theory in λ rather reliably by virtue of the asymptotic freedom.

[10] In fact, the boundary correlation function can be derived from the correlation function of the three-dimensional massive fermion by the reduction to the two-dimensional space-time. This is why the continuum spectrum with mass gap emerges.

Since S_-^v has the short correlation length of the order of the lattice spacing as we have shown, its combolutions in Eqs. (5.23) and (5.24) with the correlation function of the gauge freedom Eq. (5.25) also have short correlation lengths, even though Eq. (5.25) is logarithmically increasing function of the distance $|n|$. There is no symmetry against the mass gap. This result shows that even inside of the symmetric phase off the critical point, no light particle emerges in the boundary correlation functions. Since the overlap correlation function does not depend on the gauge freedom at all and does show the chiral spectrum[16], as we have seen, the above fact means that the entire fermion spectrum is chiral. Thus, *the disordered nature of the gauge freedom does not contradict with the chiral fermion spectrum.*

6 Summary and Discussion

In summary, we have examined the dynamical nature of the gauge degree of freedom in the lattice pure gauge model. The model is obtained in the limit $\beta = \infty$ from the four-dimensional $SU(N)$ nonabelian chiral gauge theory which is defined by the vacuum overlap and is modified in the weight of the gauge average by introducing the covariant gauge fixing term and the Faddeev-Popov determinant.

In the continuum theory, it has been noticed by Hata that the model has the dynamical features which is quite similar to the two-dimensional nonlinear sigma model. Namely, it is renormalizable in the perturbation theory and the self-coupling of the gauge freedom is asymptotically free. The severe IR divergence, which occurs in the perturbation expansion, prevents local order parameters from emerging. These dynamical features suggest that the global gauge symmetry does not break spontaneously and the gauge degree of freedom acquires mass dynamically by the dimensional transmutation.

The different feature in the lattice counterpart, which is obtained from the lattice chiral gauge theory in consideration, is that the gauge degree of freedom does not decouple from the fermion determinant and has coupling through the gauge (BRST) non-invariant piece of the complex phase of chiral determinant. What we showed first is that even in the presence of these gauge (BRST) symmetry breaking terms, the model is renormalizable at one-loop and the self-coupling of the gauge freedom is indeed asymptotically free. The IR divergence also occurs and it prevents local order parameters from emerging. Therefore, as in the continuum theory, the gauge degree of freedom is disordered at $\beta = \infty$. In this analysis, we also found that the lattice Gribov copies are no more the stationary points of the total action and should be suppressed in the gauge average.

Because of the lack of the gauge (BRST) symmetry, however, it is expected that the gauge non-invariant divergent and finite terms would be induced at some higher but finite orders. In order to clarify the effect of such terms, we next performed the perturbative analysis of the Foerster-Nielsen-Ninomiya mechanism. Namely, we introduced the explicit real gauge symmetry breaking

term, $\mathcal{O}_1$, and examined how this term would spoil the disordered nature of the gauge freedom or how small the breaking term must be in order to keep that nature. Although it is small for small λ, we found a finite region of the coupling constant of $\mathcal{O}_1$ for which the effect of the breaking term does not alter the power-law long distance behavior of the invariant two-point function of the gauge freedom. This means that, even if the gauge breaking term of the type $\mathcal{O}_1$ is induced, by tuning the counter term, we can keep the disordered nature of the gauge freedom. From this analysis, the phase structure of the pure gauge model was also argued. We also found that the model remains renormalizable with $\mathcal{O}_1$ and that the Foerster-Nielsen-Ninomiya mechanism in this perturbative framework is closely related to the IR regularization prescription of the continuum perturbation theory by the mass term for the gauge boson.

Based on these dynamical features, which is quite similar to the nonlinear sigma model in two-dimensions, and following Hata's scenario for the dynamical gauge symmetry restoration, we then assumed that the global gauge symmetry does not break spontaneously and the gauge freedom acquires mass dynamically in the pure gauge limit for the entire region of the gauge parameter.

Under this assumption, the asymptotic freedom allows us to tame the gauge fluctuation by approaching the critical point of the gauge freedom without spoiling its disordered nature. There we showed by the perturbation expansion that the spectrum in the invariant boundary correlation functions, which is free from IR divergence, have mass gap of the order of the lattice cutoff and it survives the quantum correction due to the gauge fluctuation. There is no symmetry against the spectrum mass gap. This means that no light state appear in the boundary correlation functions and they cannot be regarded as physically relevant observables.

From the overlap correlation functions, on the other hand, the gauge degree of freedom decouples completely. This is a remarkable property of the overlap correlation function. It makes it easy and clear to identify the fermion spectrum in the pure gauge limit. In fact, the correct number of free Weyl fermions emerge from this correlation function. The appearance of the poles of species doublers are suppressed by the appearance of zeros at the vanishing momenta for doublers. Therefore the entire fermion spectrum is chiral and consistent with the continuum target theory. Thus we argued that the dynamical restoration of the gauge symmetry due to the disordered gauge degree of freedom does not contradict with the chiral fermion spectrum in the vacuum overlap formulation of lattice $SU(N)$ chiral gauge theory.

Finally, let us make a few comments related to the formulation with the manifest local gauge invariance. We might start from the partition function with the manifest local gauge invariance, by introducing the explicit integration over the gauge degree of freedom:

$$Z_{inv} = \int [d\omega][dU][d\bar{c}dc] \exp\left(-\beta S_G - \frac{1}{2\alpha}\left(\bar{\nabla}^{\omega}\hat{A}\right)^2 - \bar{c}M[{}^{\omega}U]c\right) \times$$

$$\prod_{rep.} \left(\frac{\langle +|\hat{\Omega}|v+\rangle}{|\langle +|\hat{\Omega}|v+\rangle|} \langle v+|v-\rangle \frac{\langle v-|\hat{\Omega}^\dagger|-\rangle}{|\langle v-|\hat{\Omega}^\dagger|-\rangle|} \right). \tag{6.1}$$

Here the field variable ω_n takes its value in $SU(N)$ and is assumed to transform under the gauge transformation as follows:

$$\omega_n \longrightarrow \omega_n g_n^\dagger. \tag{6.2}$$

$\hat{\Omega}$ is defined by

$$\hat{\Omega} = \exp\left(\hat{a}_n^{\dagger i} \{\log \omega_n\}_i{}^j \hat{a}_{nj} \right). \tag{6.3}$$

As to the gauge link variable in the gauge fixing term and the Faddeev-Popov ghost action, they are replaced by

$${}^{\omega}U_{n\mu} = \omega_n U_{n\mu} \omega_{n+\hat{\mu}}^\dagger, \tag{6.4}$$

Accordingly, ${}^{\omega}\hat{A}_{n\mu}$ is defined from ${}^{\omega}U_{n\mu}$ by the definition of the lattice vector potential Eq. (2.15). Besides the gauge symmetry, there exists a global $SU(N)$ symmetry under the transformation which we refer as global $SU(N)_{H'}$:

$$\omega_n \longrightarrow h'\omega_n, \qquad h' \in SU(N)_{H'}. \tag{6.5}$$

At $\beta = \infty$, the partition function reduces to

$$\begin{aligned} Z_{inv} &= \int [d\omega][dg][d\bar{c}dc] \exp\left(-\frac{1}{2\alpha} \left(\bar{\nabla}^{\omega g} \hat{A} \right)^2 - \bar{c} M[\omega g] c \right) \times \\ &\qquad \prod_{rep.} \left(\frac{\langle +|\hat{\Omega}\hat{G}|+\rangle}{|\langle +|\hat{\Omega}\hat{G}|+\rangle|} \langle +|-\rangle \frac{\langle -|\hat{G}^\dagger\hat{\Omega}^\dagger|-\rangle}{|\langle -|\hat{G}^\dagger\hat{\Omega}^\dagger|-\rangle|} \right) \\ &= \int d\mu[\omega g; \alpha]. \end{aligned} \tag{6.6}$$

In this limit, there emerges another global $SU(N)$ symmetry, which we have refered as global $SU(N)_H$:

$$g_n \longrightarrow g_n h^\dagger, \qquad h \in SU(N)_H. \tag{6.7}$$

If we fix the local gauge symmetry so that

$$\omega_n = 1, \qquad \hat{\Omega} = \mathbb{1}, \tag{6.8}$$

the model reduces to the form with which we have discussed in this article. The symmetry is then reduced to

$$SU(N)_C \otimes SU(N)_H, \quad SU(N)_{H'} \sim SU(N)_C. \tag{6.9}$$

On the other hand, we can fix the local gauge symmetry so that

$$g_n = 1, \qquad \hat{G} = \mathbb{1}. \tag{6.10}$$

In this case, the symmetry is reduced to

$$SU(N)_{H'} \otimes SU(N)_C, \quad SU(N)_C \sim SU(N)_H. \tag{6.11}$$

We note that, in the context of the Wilson-Yukawa model[1, 2, 3], this way of the gauge fixing leads to the formulation in terms of the "ordinary" fermion, with ω_n to appear in the Wilson-Yukawa coupling term. There, the "C-charged" fermion is called "charged" and "H'-charged" fermion is called "neutral". With this correspondence in mind, we next clarify the correspondence of the fermion correlation functions.

The gauge covariant overlap correlation function in the pure gauge limit, which corresponds to Eq. (5.8), is given as follows:

$$\begin{aligned} \langle a_{ni} a^{\dagger\, j}_{m} \rangle_r &= \frac{1}{Z} \int d\mu[\omega g; \alpha] \times \\ & \qquad \langle + | \hat{G}^\dagger \left\{ \hat{a}_{ni} \hat{a}^{\dagger\, j}_{m} - \frac{1}{2} \delta_{nm} \delta^j_i \right\} \hat{G} | - \rangle_r \Big/ \langle + | - \rangle_r . \end{aligned} \tag{6.12}$$

In the second gauge, it reduces to the form

$$\begin{aligned} \langle a_{ni} a^{\dagger\, j}_{m} \rangle_r &= \frac{1}{Z} \int d\mu[\omega; \alpha] \times \\ & \qquad \langle + | \left\{ \hat{a}_{ni} \hat{a}^{\dagger\, j}_{m} - \frac{1}{2} \delta_{nm} \delta^j_i \right\} | - \rangle_r \Big/ \langle + | - \rangle_r . \\ &= \langle + | \left\{ \hat{a}_{ni} \hat{a}^{\dagger\, j}_{m} - \frac{1}{2} \delta_{nm} \delta^j_i \right\} | - \rangle_r \Big/ \langle + | - \rangle_r . \end{aligned} \tag{6.13}$$

Note that this is the "C-charged" ("charged") correlation function and the gauge freedom ω decouples completely from it. (Note also that this correlation function corresponds to the "colored" quark correlation function Eq. (5.3) when the gauge is fixed so that $g_n = 1$ in QCD.) This correlation function has the same structure as the "H-charged" correlation function Eq. (5.12) in the first gauge, in which, as we know, the correct number of the Weyl fermions appear. This means that in the vacuum overlap formulation, the "charged" fermions appear in the correct chiral spectrum. As for the "neutral" fermions, we have seen in the first gauge that the "C-charged" fermions cannot be observed due to the IR singularity and it means that the "H'-charged"("neutral") fermion does not appear in the physical spectrum. Therefore, there is no problem here concerning the "neutral"-ness of the fermion and the triviality of the chiral vector coupling[48]. [11]

[11] We can find in [24] (latest revised version) the similar observation. For the explicit calculation, the author refers to forthcoming papers. Note that, in the overlap formulation, once we understand that any light states do not appear in the boundary correlation functions, the spectrum of the "charged" fermion can be easily and clearly read from the overlap correlation function because the gauge degree of freedom decouples completely from it.

From these observations, we see that the several questions raised to the Wilson-Yukawa model; the absence of the "charged" fermion, the vector-like spectrum of the "neutral" fermion and the triviality of its chiral coupling to vector boson, do not apply to the lattice chiral gauge theory defined by the vacuum overlap formulation. As to the fermion number violation, it has been already demonstrated in several contexts that the overlap formulation can correctly reproduce the effect of the instanton and can describe such physical process of the fermion number violation[16, 50].

As to the Wilson-Yukawa model for the standard model itself, the possible answer for these long standing questions has been described in [24](latest revised version) in relation to the dynamics of the gauge freedom governed by the covariant-type gauge fixing term[25]. It may be interesting to reconsider the Eichten-Preskill model for the nonabelian chiral gauge theory [5, 6, 51] from the point of view of the nonabelian dynamics of the gauge degree of freedom discussed in this paper.

Acknowledgments

The author would like to thank H. Neuberger and R. Narayanan for enlightening discussions. He would like to thank H. Hata for discussion on the dynamics of the pure gauge model. He would also like to thank A. Yamada for discussion. The author express his sincere thanks to High energy physics theory group, Nuclear physics group and the computer staffs of the department of physics and astronomy of Rutgers university for their kind hospitality.

Appendix

A Expansion of the action in gauge fluctuation

In this appendix, we give the explicit formula of Eq. (3.13): the action expanded in terms of the fluctuations of the gauge freedom, ghost and anti-ghost field around a classical configuration.

$$
\begin{aligned}
S[\exp(i\pi)g, c+\xi, \bar{c}+\bar{\xi}] \;&=\; S[g,c,\bar{c}] + S_0[\pi^2/\xi, \bar{\xi}] \\
&+\; S_1[\pi^2/\xi, \bar{\xi}; \sin A] + S_{1c}[\pi, \xi/\bar{\xi}; g, c/\bar{c}] \\
&+\; S_2[\pi^2/\xi, \bar{\xi}; \sin^2 A, \cos A - 1] + S_{2c}[\pi^2; g, c, \bar{c}] \\
&+\; i\,\Delta S_{WZW}[\pi^2; g],
\end{aligned} \tag{A.1}
$$

where

$$\begin{aligned}
S[g,c,\bar{c}] &= -\sum_{n,a}\frac{1}{2\alpha}\left(\bar{\nabla}_\mu \hat{A}^a_{n\mu}\right)^2 - \sum_{nm,ab}\bar{c}^a_n \hat{M}^{ab}_{nm}[g_n g^\dagger_{n+\hat{\mu}}]c^b_m \\
&+ i\Delta\Gamma_{WZW}[g], \qquad (A.2)\\
S_0[\pi^2/\xi,\bar{\xi}] &= -\sum_{n,a}\frac{1}{2}\left(\nabla^2\pi^a_n\right)^s + \sum_{n,a}\bar{\xi}^a_n\nabla^2\xi^a_n, \qquad (A.3)\\
S_1[\pi^2/\xi,\bar{\xi};A] &= -\sum_{n\mu}\frac{1}{2}f^{abc}\nabla^2\pi^a_n\left(\pi^b_{n+\hat{\mu}}\hat{A}^c_{n\mu} - \pi^b_{n-\hat{\mu}}\hat{A}^c_{n-\hat{\mu},\mu}\right)\\
&+ \sum_n \frac{1}{2}f^{abc}\bar{\xi}^a_n\bar{\nabla}_\mu\left\{\left(\xi^b_n+\xi^b_{n+\hat{\mu}}\right)\hat{A}^c_{n\mu}\right\}\\
&= \sum_{n\mu}\frac{1}{2}\bar{\nabla}_\mu\hat{A}^a_{n\mu}f^{abc}\pi^b_n\nabla^2\pi^c_n\\
&- \sum_{n\mu}\frac{1}{2}\nabla^2\pi^a_n f^{abc}\left(\nabla_\mu\pi^b_n\hat{A}^c_{n\mu} + \bar{\nabla}_\mu\pi^b_n\hat{A}^c_{n-\hat{\mu},\mu}\right)\\
&+ \sum_n \frac{1}{2}f^{abc}\bar{\xi}^a_n\bar{\nabla}_\mu\left\{\left(\xi^b_n+\xi^b_{n+\hat{\mu}}\right)\hat{A}^c_{n\mu}\right\}, \qquad (A.4)\\
S_2[\pi^2/\xi,\bar{\xi};A^2] &= \sum_n \frac{1}{2}\bar{\nabla}_\nu\hat{A}^a_{n\nu}\times\\
&\quad \bar{\nabla}_\mu\{\mathrm{Tr}\left[(\pi_{n+\hat{\mu}}[\pi_{n+\hat{\mu}},T^a]\right.\\
&\qquad +\pi_n[\pi_n,T^a]+[T^a,\pi_{n+\hat{\mu}}]\pi_{n+\hat{\mu}}\\
&\qquad \left.+[T^a,\pi_n]\pi_n + 2\nabla_\mu\pi_n T^a\nabla_\mu\pi_n)\sin A_{n\mu}\right]\}\\
&- \sum_n \frac{1}{8}f^{abc}\left(2\pi^b_n\bar{\nabla}_\nu\hat{A}^c_{n\nu} + \nabla_\nu\pi^b_n\hat{A}^c_{n\nu} + \bar{\nabla}_\nu\pi^b_n\hat{A}^c_{n-\hat{\nu},\nu}\right)\times\\
&\qquad f^{ade}\left(2\pi^d_n\bar{\nabla}_\mu\hat{A}^e_{n\mu} + \nabla_\mu\pi^d_n\hat{A}^e_{n\mu} + \bar{\nabla}_\mu\pi^d_n\hat{A}^e_{n-\hat{\mu},\mu}\right)\\
&- \sum_n \nabla^2\pi^a_n\bar{\nabla}_\mu\left\{\mathrm{Tr}\left[\{T^a,T^b\}\left(\cos A_{n\mu}-1\right)\right]\nabla_\mu\pi^b_n\right\}\\
&+ \sum_n \bar{\xi}^a_n\bar{\nabla}_\mu\left\{\mathrm{Tr}\left[\{T^a,T^b\}\left(\cos A_{n\mu}-1\right)\right]\nabla_\mu\xi^b_n\right\}, \qquad (A.5)\\
S_{1c}[\pi,\xi/\bar{\xi};g,c/\bar{c}] &= \sum_{n\mu} i\,\bar{\xi}^a_n\bar{\nabla}_\mu\left\{\mathrm{Tr}\left[\left(T^aT^b\pi_n - \pi_{n+\hat{\mu}}T^aT^b\right)g_n g^\dagger_{n+\hat{\mu}}\right.\right.\\
&\qquad \left. + \left(T^bT^a\pi_{n+\hat{\mu}} - \pi_n T^bT^a\right)g_{n+\hat{\mu}}g^\dagger_n\right]c^b_n\\
&\qquad -\mathrm{Tr}\left[\left(T^bT^a\pi_n - \pi_{n+\hat{\mu}}T^bT^a\right)g_n g^\dagger_{n+\hat{\mu}}\right.\\
&\qquad \left.\left. + \left(T^aT^b\pi_{n+\hat{\mu}} - \pi_n T^aT^b\right)g_{n+\hat{\mu}}g^\dagger_n\right]c^b_{n+\hat{\mu}}\right\}\\
&+ \sum_{n\mu} i\,\bar{c}^a_n\bar{\nabla}_\mu\left\{\mathrm{Tr}\left[\left(T^aT^b\pi_n - \pi_{n+\hat{\mu}}T^aT^b\right)g_n g^\dagger_{n+\hat{\mu}}\right.\right.\\
&\qquad\qquad \left. + \left(T^bT^a\pi_{n+\hat{\mu}} - \pi_n T^bT^a\right)g_{n+\hat{\mu}}g^\dagger_n\right]\xi^b_n
\end{aligned}$$

$$
\begin{aligned}
& -\mathrm{Tr}\left[\left(T^bT^a\pi_n - \pi_{n+\hat{\mu}}T^bT^a\right) g_n g^\dagger_{n+\hat{\mu}} \right. \\
& \left. + \left(T^aT^b\pi_{n+\hat{\mu}} - \pi_n T^aT^b\right) g_{n+\hat{\mu}} g^\dagger_n\right] \xi^b_{n+\hat{\mu}} \Big\} ,
\end{aligned} \tag{A.6}
$$

$$
\begin{aligned}
S_{2c}[\pi^2; g, c, \bar{c}] \;=\; & \sum_{n\mu} \frac{1}{2} \nabla_\mu \bar{c}^a_n \times \\
& \Big\{ \mathrm{Tr}\left[\left(T^aT^b\pi^2_n - 2\pi_{n+\hat{\mu}}T^aT^b\pi_n + \pi^2_{n+\hat{\mu}}T^aT^b\right) g_n g^\dagger_{n+\hat{\mu}} \right. \\
& \left. + \left(T^bT^a\pi^2_{n+\hat{\mu}} - 2\pi_n T^bT^a\pi_{n+\hat{\mu}} + \pi^2_n T^bT^a\right) g_{n+\hat{\mu}} g^\dagger_n\right] c^b_n \\
& -\mathrm{Tr}\left[\left(T^bT^a\pi^2_n - 2\pi_{n+\hat{\mu}}T^bT^a\pi_n + \pi^2_{n+\hat{\mu}}T^bT^a\right) g_n g^\dagger_{n+\hat{\mu}} \right. \\
& \left. + \left(T^aT^b\pi^2_{n+\hat{\mu}} - 2\pi_n T^aT^b\pi_{n+\hat{\mu}} + \pi^2_n T^aT^b\right) g_{n+\hat{\mu}} g^\dagger_n\right] \\
& \qquad c^b_{n+\hat{\mu}} \Big\} .
\end{aligned} \tag{A.7}
$$

B Local operators of dimensions less than five

The possible local operators of dimensions less than five can be listed up based on the global $SU(N)$ gauge symmetry, the hyper-cubic symmetry, the CP invariance, the ghost number conservation and the anti-ghost shift symmetry[22]. The complete list for the $SU(N)$ gauge group is given in this appendix.

$$
\mathcal{O}_0 = -\frac{1}{2\alpha}\sum_n \left(\bar{\nabla}_\mu A^a_{n\mu}\right)^2 - \sum_n \bar{c}^a_n \bar{\nabla}_\mu \left(-\nabla_\mu c^a_n + \frac{1}{2} f^{abc}(c^b_n + c^b_{n+\hat{\mu}}) A^c_\mu \right), \tag{B.1}
$$

$$
\mathcal{O}_1 = -\frac{1}{2}\sum_{n\mu} A^a_{n\mu} A^a_{n\mu}, \tag{B.2}
$$

$$
\mathcal{O}_2 = -\frac{1}{2}\sum_{n\mu\nu} \bar{\nabla}_\mu A^a_{n\nu} \bar{\nabla}_\mu A^a_{n\nu}, \tag{B.3}
$$

$$
\mathcal{O}_3 = -\frac{1}{2}\sum_{n\mu} \bar{\nabla}_\mu A^a_{n\mu} \bar{\nabla}_\mu A^a_{n\mu}, \tag{B.4}
$$

$$
\mathcal{O}_4 = \sum_{n\mu} \bar{c}^a_n \bar{\nabla}_\mu \nabla_\mu c^a_n, \tag{B.5}
$$

$$
\mathcal{O}_5 = -\frac{1}{2}\sum_{n\mu} \bar{c}^a_n \bar{\nabla}_\mu \left(f^{abc}(c^b_n + c^b_{n+\hat{\mu}}) A^c_\mu \right), \tag{B.6}
$$

$$
\mathcal{O}_6 = \sum_{n\mu\nu} f^{abc} \bar{\nabla}_\mu A^a_{n\nu} A^b_{n\mu} A^c_{n\nu}, \tag{B.7}
$$

$$
\mathcal{O}_7 = \sum_{n\mu\nu} A^a_{n\mu} A^a_{n\mu} A^b_{n\nu} A^b_{n\nu}, \tag{B.8}
$$

$$\mathcal{O}_8 = \sum_{n\mu\nu} A^a_{n\mu} A^b_{n\mu} A^a_{n\nu} A^b_{n\nu}, \tag{B.9}$$

$$\mathcal{O}_9 = \sum_{n\mu\nu} f^{ace} f^{bde} A^a_{n\mu} A^b_{n\mu} A^c_{n\nu} A^d_{n\nu}, \tag{B.10}$$

$$\mathcal{O}_{10} = \sum_{n\mu\nu} d^{abe} d^{cde} A^a_{n\mu} A^b_{n\mu} A^c_{n\nu} A^d_{n\nu}, \tag{B.11}$$

$$\mathcal{O}_{11} = \sum_{n\mu\nu} d^{ace} d^{bde} A^a_{n\mu} A^b_{n\mu} A^c_{n\nu} A^d_{n\nu}, \tag{B.12}$$

$$\mathcal{O}_{12} = \sum_{n\mu} A^a_{n\mu} A^a_{n\mu} A^b_{n\mu} A^b_{n\mu}, \tag{B.13}$$

$$\mathcal{O}_{13} = \sum_{n\mu} d^{abe} d^{cde} A^a_{n\mu} A^b_{n\mu} A^c_{n\mu} A^d_{n\mu}. \tag{B.14}$$

C Contribution of the Wess-Zumino-Witten term

$i\Delta S_{WZW}$ in Eq. (3.13) is the contribution from the Wess-Zumino-Witten term. This is evaluated from Eq. (3.17),

$$\begin{aligned} e^{i\Delta S_{WZW}[\pi;g]} &= e^{i\Delta\Gamma_{WZW}[\exp(i\lambda\pi)g]-i\Delta\Gamma_{WZW}[g]} \\ &= \prod_{rep.} \left(\frac{\langle +|\hat{\Pi}\hat{G}|+\rangle}{|\langle +|\hat{\Pi}\hat{G}|+\rangle|} \frac{\langle -|\hat{G}^\dagger\hat{\Pi}^\dagger|-\rangle}{|\langle -|\hat{G}^\dagger\hat{\Pi}^\dagger|-\rangle|} \Bigg/ \frac{\langle +|\hat{G}|+\rangle}{|\langle +|\hat{G}|+\rangle|} \frac{\langle -|\hat{G}^\dagger|-\rangle}{|\langle -|\hat{G}^\dagger|-\rangle|} \right) \\ &= \prod_{rep.} \left(\frac{\langle +|\hat{\Pi}|v+\rangle}{|\langle +|\hat{\Pi}|v+\rangle|} \frac{\langle v-|\hat{\Pi}^\dagger|-\rangle}{|\langle v-|\hat{\Pi}^\dagger|-\rangle|} \Bigg/ \frac{\langle +|v+\rangle}{|\langle +|v+\rangle|} \frac{\langle v-|-\rangle}{|\langle v-|-\rangle|} \right). \end{aligned} \tag{C.1}$$

$\hat{\Pi}$ is the operator of the gauge transformation due to π given by:

$$\hat{\Pi} = \exp\left(\hat{a}^{\dagger i}_n \{i\lambda\pi_n\}_i{}^j \hat{a}_{nj} \right). \tag{C.2}$$

$|v+\rangle$ and $|v-\rangle$ are given in this case by the vacua of the second-quantized Hamiltonians with the pure gauge link variable which satisfies the classical equation of motion and is parameterized by the pure gauge vector potential $A_{n\mu}$ as in Eq. (3.11),

$$g_n g^\dagger_{n+\hat{\mu}} = \exp\left(iA_{n\mu}\right). \tag{C.3}$$

We can evaluate these vacua in the expansion in terms of $A_{n\mu}$ using the Hamiltonian perturbation theory. Then $i\Delta S_{WZW}$ is obtained in the following expansion,

$$i\,\Delta S_{WZW}[\pi;g] = \sum_{k=1}^{\infty} i\,\Delta S_{k\,WZW}[\pi;A^k]. \tag{C.4}$$

This result is further expanded in terms of π. The linear term in π gives the contribution to the classical equation. (Note that the formula of this contribution

given in Eq. (3.4) is evaluated directly from the first expression of Eq. (3.17).) The quadratic term in π is what we need for the one-loop calculation by the background field method.

C.1 Background vacua

The second quantized Hamiltonians Eq. (2.7) with the pure gauge link variable of the classical solution are divided into the free part and the perturbative part. (Here after we omit the suffixes of the signature of masses, $\pm$, as long as it does not introduce any confusions.)

$$\hat{H}\left[g_n g^{\dagger}_{n+\hat{\mu}}\right] = \hat{H}_0 + \hat{V}\left[g_n g^{\dagger}_{n+\hat{\mu}}\right], \tag{C.5}$$

where

$$\hat{V} = \hat{V}_1 + \hat{V}_2, \tag{C.6}$$

$$\begin{aligned} V_1(n,m)^j_i &= i\gamma_5\left[\left(\frac{\gamma_\mu - \mathbb{1}}{2}\right)\delta_{n+\hat{\mu},m}\sin A_{n\mu} + \left(\frac{\gamma_\mu + \mathbb{1}}{2}\right)\delta_{n,m+\hat{\mu}}\sin A_{m\mu}\right] \\ &= \int \frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\, e^{iqn-ipm} \times \\ &\quad \gamma_5\left[i\gamma_\mu \cos\left(\frac{q_\mu + p_\mu}{2}\right) + \sin\left(\frac{q_\mu + p_\mu}{2}\right)\right]\sin A_\mu(q-p), \end{aligned} \tag{C.7}$$

$$\begin{aligned} V_2(n,m)^j_i &= \gamma_5\left[\left(\frac{\gamma_\mu - \mathbb{1}}{2}\right)\delta_{n+\hat{\mu},m}\left(\cos A_{n\mu} - 1\right)\right. \\ &\qquad \left. - \left(\frac{\gamma_\mu + \mathbb{1}}{2}\right)\delta_{n,m+\hat{\mu}}\left(\cos A_{m\mu} - 1\right)\right] \\ &= \int \frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\, e^{iqn-ipm} \times \\ &\quad \gamma_5\left[i\gamma_\mu \sin\left(\frac{q_\mu + p_\mu}{2}\right) - \cos\left(\frac{q_\mu + p_\mu}{2}\right)\right]\left(\cos A_\mu(q-p) - 1\right). \end{aligned} \tag{C.8}$$

We denote the eigenstates of the free Hamiltonian as

$$\hat{H}_0|n\rangle = E_n|n\rangle, \quad (n = 0, 1, \ldots). \tag{C.9}$$

We also denote the one-particle state normalized eigenvectors of Hamiltonian by $u(p,s)$ and $v(p,s)$ $(s = 1, 2)$:

$$H_{0\,nm}\left(e^{ipm}u(p,s)\right) = +\lambda(p)\left(e^{ipn}u(p,s)\right), \tag{C.10}$$

$$H_{0,nm}\left(e^{ipm}v(p,s)\right) = -\lambda(p)\left(e^{ipn}v(p,s)\right), \tag{C.11}$$

and introduce the projection operators to the positive and negative eigenvectors by

$$
\begin{aligned}
S^u(p) &= \sum_s u(p,s)u(p,s)^\dagger, && \text{(C.12)}\\
S^v(p) &= \sum_s v(p,s)v(p,s)^\dagger. && \text{(C.13)}
\end{aligned}
$$

The ground state is evaluated up to the second order of the perturbation $\hat{V}$ as follows:

$$
\begin{aligned}
|v\rangle &= |0\rangle\left(1-\frac{1}{2}\sum_{n>0}\langle 0|V|n\rangle\frac{1}{(E_0-E_n)^2}\langle n|V|0\rangle\right)\\
&+ \sum_{n>0}|n\rangle\frac{1}{E_0-E_n}\langle n|V|0\rangle\\
&+ \sum_{m,n>0}|n\rangle\frac{1}{E_0-E_n}\langle n|V|m\rangle\frac{1}{E_0-E_m}\langle m|V|0\rangle\\
&- \sum_{n>0}|n\rangle\frac{1}{(E_0-E_n)^2}\langle n|V|0\rangle\langle 0|V|0\rangle\\
&+ \mathcal{O}(V^3). \qquad \text{(C.14)}
\end{aligned}
$$

Since our perturbation $\hat{V}$ is bilinear in the fermion operators, only the two-particle states can contribute as intermediate states. The fermion number is also conserved by the perturbation and the total fermion number of the intermediate states always vanishes because we are considering the vacuum to vacuum transition amplitude. The intermediate-state projection operator is then evaluated as

$$
\begin{aligned}
\sum_{\text{two particle}}|n\rangle\frac{1}{E_0-E_n}\langle n| &= -\int\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{e^{iq(n-n')+ip(m-m')}}{\lambda(q)+\lambda(p)}\\
&\quad\Big\{S^u(p)_{\alpha\beta}S^v(q)_{\gamma\delta}\,a^\dagger_{m\alpha}a_{n\delta}|0\rangle\langle 0|a^\dagger_{n'\gamma}a_{m'\beta}\\
&\quad+S^v(p)_{\alpha\beta}S^u(q)_{\gamma\delta}\,a_{m\beta}a^\dagger_{n\gamma}|0\rangle\langle 0|a_{n'\delta}a^\dagger_{m'\alpha}\Big\}. \qquad \text{(C.15)}
\end{aligned}
$$

Matrix elements which appear in the above formula of the ground state perturbation can be evaluated as follows:

$$
\begin{aligned}
\langle 0|\hat{V}|0\rangle &= \mathrm{Tr}\left\{V(s,t)S^v(t-s)\right\}, && \text{(C.16)}\\
\langle 0|a_{m\alpha}a^\dagger_{n\beta}\hat{V}|0\rangle &= +S^v(m-n)_{\alpha\beta}\mathrm{Tr}\left\{V\cdot S^v\right\}-S^u\cdot V\cdot S^v(m,n)_{\alpha\beta}, && \text{(C.17)}\\
\langle 0|a^\dagger_{m\alpha}a_{n\beta}\hat{V}|0\rangle &= -S^v(n-m)_{\beta\alpha}\mathrm{Tr}\left\{V\cdot S^v\right\}+S^u\cdot V\cdot S^v(n,m)_{\beta\alpha}, && \text{(C.18)}
\end{aligned}
$$

$$
\begin{aligned}
&\langle 0|a_{m_1\alpha_1}a^\dagger_{n_1\beta_1}\hat{V}a_{m_2\alpha_2}a^\dagger_{n_2\beta_2}|0\rangle \\
&= +S^u(m_1-n_1)_{\alpha_1\beta_1}S^u(m_2-n_2)_{\alpha_2\beta_2}\,\mathrm{Tr}\left\{V\cdot S^v\right\} \\
&+S^u(m_1-n_2)_{\alpha_1\beta_2}S^v(m_2-n_1)_{\alpha_2\beta_1}\,\mathrm{Tr}\left\{V\cdot S^v\right\} \\
&-S^u(m_1-n_1)_{\alpha_1\beta_1}S^v\cdot V\cdot S^u(m_2,n_2)_{\alpha_2\beta_2} \\
&-S^u(m_1-n_2)_{\alpha_1\beta_2}S^v\cdot V\cdot S^v(m_2,n_1)_{\alpha_2\beta_1} \\
&+S^u\cdot V\cdot S^u(m_1,n_2)_{\alpha_1\beta_2}S^v(m_2-n_1)_{\alpha_2\beta_1} \\
&-S^u\cdot V\cdot S^u(m_1,n_1)_{\alpha_1\beta_1}S^u(m_2-n_2)_{\alpha_2\beta_2},
\end{aligned}
\tag{C.19}
$$

$$
\begin{aligned}
&\langle 0|a^\dagger_{n_1\beta_1}a_{m_1\alpha_1}\hat{V}a_{m_2\alpha_2}a^\dagger_{n_2\beta_2}|0\rangle \\
&= +S^v(m_1-n_1)_{\alpha_1\beta_1}S^u(m_2-n_2)_{\alpha_2\beta_2}\,\mathrm{Tr}\left\{V\cdot S^v\right\} \\
&-S^u(m_1-n_2)_{\alpha_1\beta_2}S^v(m_2-n_1)_{\alpha_2\beta_1}\,\mathrm{Tr}\left\{V\cdot S^v\right\} \\
&-S^v(m_1-n_1)_{\alpha_1\beta_1}S^v\cdot V\cdot S^u(m_2,n_2)_{\alpha_2\beta_2} \\
&+S^u(m_1-n_2)_{\alpha_1\beta_2}S^v\cdot V\cdot S^v(m_2,n_1)_{\alpha_2\beta_1} \\
&-S^u\cdot V\cdot S^u(m_1,n_2)_{\alpha_1\beta_2}S^v(m_2-n_1)_{\alpha_2\beta_1} \\
&+S^u\cdot V\cdot S^v(m_1,n_1)_{\alpha_1\beta_1}S^u(m_2-n_2)_{\alpha_2\beta_2}.
\end{aligned}
\tag{C.20}
$$

Using these results, we obtain

$$
\begin{aligned}
&\sum_{\text{two particle}}|n\rangle\frac{1}{E_0-E_n}\langle n|\hat{V}|0\rangle \\
&=\int\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\,\frac{2}{\lambda(q)+\lambda(p)}e^{iqn}\left[S^u(q)V(q;p)S^v(p)\right]_{\alpha\beta}e^{-ipm}\,\times \\
&\quad\frac{1}{2}\left\{a_{m\beta}a^\dagger_{n\alpha}-a^\dagger_{n\alpha}a_{m\beta}\right\}|0\rangle.
\end{aligned}
\tag{C.21}
$$

$$
\begin{aligned}
&\sum_{\text{two particle}}|n\rangle\frac{1}{E_0-E_n}\langle n|V|m\rangle\frac{1}{E_0-E_m}\langle m|V|0\rangle \\
&=\int\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\,\frac{2}{\lambda(q)+\lambda(k)}\frac{2}{\lambda(k)+\lambda(p)} \\
&\quad e^{iqn}\left[S^u(q)V(q;k)(\,S^v(k)-S^u(k)\,)V(k;p)S^v(p)\right]_{\alpha\beta}e^{-ipm}\,\times \\
&\quad\frac{1}{2}\left\{a_{m\beta}a^\dagger_{n\alpha}-a^\dagger_{n\alpha}a_{m\beta}\right\}|0\rangle.
\end{aligned}
\tag{C.22}
$$

$$
\begin{aligned}
&\sum_{\text{two particle}}|n\rangle\frac{1}{(E_0-E_n)^2}\langle n|V|0\rangle\langle 0|V|0\rangle \\
&=\int\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\,\frac{4}{\left(\lambda(q)+\lambda(p)\right)^2}
\end{aligned}
$$

$$\begin{aligned}&\mathrm{Tr}\left\{V(k;k)S^v(k)\right\}e^{iqn}\left[S^u(q)V(q;p)S^v(p)\right]_{\alpha\beta}e^{-ipm}\times\\&\frac{1}{2}\left\{a_{m\beta}a^\dagger_{n\alpha}-a^\dagger_{n\alpha}a_{m\beta}\right\}|0\rangle.\end{aligned} \tag{C.23}$$

$$\begin{aligned}&\frac{1}{2}\sum_{\text{two particle}}\langle 0|V|n\rangle\frac{1}{(E_0-E_n)^2}\langle n|V|0\rangle\\&=-\frac{1}{2}\int\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{4}{(\lambda(q)+\lambda(p))^2}\mathrm{Tr}\left\{S^u(q)V(q;p)S^v(p)V(p;q)\right\}.\end{aligned} \tag{C.24}$$

C.2 Overlap with insertion of $\hat{\Pi}$

Given the expression of the background vacuum, its overlap to the free vacuum with the insertion of the operator of the gauge fluctuation $\hat{\Pi}$ is evaluated as

$$\begin{aligned}&\langle 0|\hat{\Pi}|v\rangle\\&=\langle 0|\hat{\Pi}|0\rangle\times\\&\left(1+\frac{1}{2}\int\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{4}{(\lambda(q)+\lambda(p))^2}\mathrm{Tr}\left\{S^u(q)V(q;p)S^v(p)V(p;q)\right\}\right)\\&+\langle 0|\hat{\Pi}\,\frac{1}{2}\left\{a_{m\beta}a^\dagger_{n\alpha}-a^\dagger_{n\alpha}a_{m\beta}\right\}|0\rangle\times\\&\left\{\int\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{2}{\lambda(q)+\lambda(p)}e^{iqn}\left[S^u(q)V(q;p)S^v(p)\right]_{\alpha\beta}e^{-ipm}\right.\\&+\int\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{2}{\lambda(q)+\lambda(k)}\frac{2}{\lambda(k)+\lambda(p)}\\&\qquad e^{iqn}\left[S^u(q)V(q;k)(\,S^v(k)-S^u(k)\,)V(k;p)S^v(p)\right]_{\alpha\beta}e^{-ipm}\\&-\int\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{4}{(\lambda(q)+\lambda(p))^2}\\&\qquad\left.\mathrm{Tr}\left\{V(k;k)S^v(k)\right\}e^{iqn}\left[S^u(q)V(q;p)S^v(p)\right]_{\alpha\beta}e^{-ipm}\right\}+\mathcal{O}(\hat{V}^3).\end{aligned} \tag{C.25}$$

The correlation function in the above equation

$$\langle 0|\hat{\Pi}\frac{1}{2}\left\{a_{m\beta}a^\dagger_{n\alpha}-a^\dagger_{n\alpha}a_{m\beta}\right\}|0\rangle/\langle 0|\hat{\Pi}|0\rangle \tag{C.26}$$

is nothing but the boundary correlation function calculated in [19] and is given by

$$\frac{1}{2}\delta_{mn}\delta^j_i-S^v[\exp(-i\lambda\pi)](m,n)^0_i\ \exp(i\lambda\pi)^j_o, \tag{C.27}$$

where

$$
\begin{aligned}
&S^v[\exp(-i\lambda\pi)](n,m)_i^j \\
&\equiv \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\, e^{ipm}e^{-iqn} \times \\
&\quad v(p,s)\left[v^\dagger(q,s')e^{-iqr}\left(\exp(i\lambda\pi_r)_j^i\right)e^{ipr}v(p,s)\right]^{-1}_{(p,s,i)(q,s',j)} v^\dagger(q,s').
\end{aligned}
\tag{C.28}
$$

It has the following expansion in terms of π (λ).

$$
\begin{aligned}
&\langle 0|\hat{\Pi}\frac{1}{2}\left\{a_{m\beta}a^\dagger_{n\alpha} - a^\dagger_{n\alpha}a_{m\beta}\right\}|0\rangle/\langle 0|\hat{\Pi}|0\rangle \\
&= \frac{1}{2}\delta_{mn} - S^v(m-n) \\
&-\sum_r S^v(m-r)(i\lambda\pi_r)S^v(r-n) - S^v(m-n)(-i\lambda\pi_n) \\
&-\sum_r S^v(m-r)\frac{1}{2!}(i\lambda\pi_r)^2 S^v(r-n) \\
&+\sum_{r,l} S^v(n-r)(i\lambda\pi_r)S^v(r-l)(i\lambda\pi_l)S^v(l-m) \\
&-\sum_r S^v(m-r)(i\lambda\pi_r)S^v(r-n)(-i\lambda\pi_n) - S^v(m-n)\frac{1}{2!}(-i\lambda\pi_n)^2 \\
&+\mathcal{O}(\lambda^3).
\end{aligned}
\tag{C.29}
$$

C.3 Expressions of $i\Delta S_{1\,WZW}[\pi^2;A]$ and $i\Delta S_{2\,WZW}[\pi^2;A^2]$

By inserting the quadratic terms of the above expansion into Eq. (C.25) and extracting the imaginary part, we finally obtain

$$
\begin{aligned}
&i\Delta S_{1\,WZW}[\pi^2;A] \\
&= \frac{1}{2}\sum_{rep.}\mathrm{Tr}\Bigg[\bar{V}_{1+}[A]\times \\
&\qquad\qquad \left\{\frac{1}{2}\left(S^v_+\pi^2S^u_+ - S^u_+\pi^2S^v_+\right) - \left(S^v_+\pi S^v_+\pi S^u_+ - S^u_+\pi S^v_+\pi S^v_+\right)\right\}\Bigg] \\
&-\frac{1}{2}\sum_{rep.}\mathrm{Tr}\Bigg[\bar{V}_{1-}[A]\times \\
&\qquad\qquad \left\{\frac{1}{2}\left(S^v_-\pi^2S^u_- - S^u_-\pi^2S^v_-\right) - \left(S^v_-\pi S^v_-\pi S^u_- - S^u_-\pi S^v_-\pi S^v_-\right)\right\}\Bigg],
\end{aligned}
\tag{C.30}
$$

$$
\begin{aligned}
&i\Delta S_{2\,WZW}[\pi^2;A] \\
&= \frac{1}{2}\sum_{rep.}\mathrm{Tr}\Big[\left(\bar{V}_{2+}[A^2] + \bar{V}_{1+}[A](S^v_+ - S^u_+)\bar{V}_{1+}[A]\right)\times
\end{aligned}
$$

$$\left\{\frac{1}{2}\left(S_+^v\pi^2 S_+^u - S_+^u\pi^2 S_+^v\right) - \left(S_+^v\pi S_+^v\pi S_+^u - S_+^u\pi S_+^v\pi S_+^v\right)\right\}\Bigg]$$

$$-\frac{1}{2}\sum_{rep.}\mathrm{Tr}\left[\left(\bar{V}_{2-}[A^2] + \bar{V}_{1-}[A](S_-^v - S_-^u)\bar{V}_{1-}[A]\right)\times\right.$$

$$\left.\left\{\frac{1}{2}\left(S_-^v\pi^2 S_-^u - S_-^u\pi^2 S_-^v\right) - \left(S_-^v\pi S_-^v\pi S_-^u - S_-^u\pi S_-^v\pi S_-^v\right)\right\}\right], \tag{C.31}$$

where

$$\begin{aligned}\bar{V}_{1\pm}[A]_{nm}{}_i^j &= \int \frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\, e^{iqn-ipm}\frac{2}{\lambda_\pm(q)+\lambda_\pm(p)}\times \\ &\quad \gamma_5\left[i\gamma_\mu\cos\left(\frac{q_\mu+p_\mu}{2}\right)+\sin\left(\frac{q_\mu+p_\mu}{2}\right)\right]\,\tilde{A}_\mu(q-p)_i^j,\end{aligned} \tag{C.32}$$

$$\begin{aligned}\bar{V}_{2\pm}[A^2]_{nm}{}_i^j &= \int \frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\, e^{iqn-ipm}\frac{2}{\lambda_\pm(q)+\lambda_\pm(p)}\times \\ &\quad \gamma_5\left[-i\gamma_\mu\sin\left(\frac{q_\mu+p_\mu}{2}\right)+\cos\left(\frac{q_\mu+p_\mu}{2}\right)\right]\times \\ &\quad\quad \int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}(2\pi)^4\delta(q-p-k_1-k_2)\,\frac{1}{2!}(\tilde{A}_\mu(k_1)\tilde{A}_\mu(k_2))_i^j,\end{aligned} \tag{C.33}$$

and

$$\begin{aligned}&S_\pm^u(n-m) \\ &= \int\frac{d^4p}{(2\pi)^4}\,e^{ip(n-m)}\left\{\frac{1}{2}+\frac{\gamma_5\left[i\gamma_\mu\sin p_\mu+\sum_\mu(1-\cos p_\mu)\pm m_0\right]}{2\lambda_\pm(p)}\right\},\end{aligned} \tag{C.34}$$

$$\begin{aligned}&S_\pm^v(n-m) \\ &= \int\frac{d^4p}{(2\pi)^4}\,e^{ip(n-m)}\left\{\frac{1}{2}-\frac{\gamma_5\left[i\gamma_\mu\sin p_\mu+\sum_\mu(1-\cos p_\mu)\pm m_0\right]}{2\lambda_\pm(p)}\right\},\end{aligned} \tag{C.35}$$

$$\lambda_\pm(p) = \sqrt{\sum_\mu \sin^2 p_\mu + \left(\sum_\mu(1-\cos p_\mu)\pm m_0\right)^2}. \tag{C.36}$$

It is not difficult to check that these two terms change the sign under parity and charge conjugation transformations and invariant under CP transformation. There is no reason based on symmetry for these terms to vanish at finite lattice cutoff.

C.4 Properties of $i\Delta S_{1\,WZW}[\pi^2;A]$ and $i\Delta S_{2\,WZW}[\pi^2;A^2]$

We first show that the leading term $i\Delta S_{1\,WZW}$ vanishes identically,

$$i\,\Delta S_{1\,WZW}[\pi^2;A]=0. \tag{C.37}$$

It is explicitly evaluated as follows:

$$\begin{aligned} i\Delta S_{1\,WZW}[\pi^2;A] &= \sum_{rep.}\sum_{n,l_1,l_2}\mathrm{Tr}\left\{A_{n\mu}\pi_{l_1}\pi_{l_2}\right\}\times \\ &\quad \int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,e^{ik_1(l_1-m)+ik_2(l_2-m)}\,\Gamma_{1\,WZW\,\mu}(k_1,k_2), \end{aligned} \tag{C.38}$$

where $(p=-k_1-k_2)$

$$\begin{aligned} &\Gamma_{1\,WZW\,\mu}(k_1,k_2) \\ &=\frac{1}{2}\lambda^2\int\frac{d^4l}{(2\pi)^4}\frac{2}{\lambda_\pm(l)+\lambda_\pm(l+p)}\times \\ &\quad \mathrm{Tr}\left[\gamma_5\left[i\gamma_\mu\cos\left(l_\mu+\frac{p_\mu}{2}\right)+\sin\left(l_\mu+\frac{p_\mu}{2}\right)\right]\times\right. \\ &\qquad \left\{\left(\frac{1}{2}-\frac{\gamma_5\left[i\gamma_\rho\sin l_\rho+\sum_\rho(1-\cos l_\rho)\pm m_0\right]}{2\lambda_\pm(l)}\right)\times\right. \\ &\qquad\quad \left(\frac{\gamma_5\left[i\gamma_\sigma\sin(l_\sigma-k_{2\sigma})+\sum_\sigma(1-\cos(l_\sigma-k_{2\sigma}))\pm m_0\right]}{2\lambda_\pm(l-k_2)}\right)\times \\ &\qquad\quad \left(\frac{1}{2}+\frac{\gamma_5\left[i\gamma_\lambda\sin(l_\lambda+p_\lambda)+\sum_\lambda(1-\cos(l_\lambda+p_\lambda))\pm m_0\right]}{2\lambda_\pm(l+p)}\right) \\ &\qquad -\left(\frac{1}{2}+\frac{\gamma_5\left[i\gamma_\rho\sin l_\rho+\sum_\rho(1-\cos l_\rho)\pm m_0\right]}{2\lambda_\pm(l)}\right)\times \\ &\qquad\quad \left(\frac{\gamma_5\left[i\gamma_\sigma\sin(l_\sigma-k_{2\sigma})+\sum_\sigma(1-\cos(l_\sigma-k_{2\sigma}))\pm m_0\right]}{2\lambda_\pm(l-k_2)}\right)\times \\ &\qquad\quad \left.\left.\left(\frac{1}{2}-\frac{\gamma_5\left[i\gamma_\lambda\sin(l_\lambda+p_\lambda)+\sum_\lambda(1-\cos(l_\lambda+p_\lambda))\pm m_0\right]}{2\lambda_\pm(l+p)}\right)\right\}\right] \\ &=\frac{1}{2}\lambda^2\int\frac{d^4l}{(2\pi)^4}\frac{2}{\lambda_\pm(l)+\lambda_\pm(l+p)}\times \\ &\quad \mathrm{Tr}\left[\gamma_5\left[i\gamma_\mu\cos\left(l_\mu+\frac{p_\mu}{2}\right)+\sin\left(l_\mu+\frac{p_\mu}{2}\right)\right]\times\right. \\ &\qquad \left\{\left(\frac{\gamma_5\left[i\gamma_\sigma\sin(l_\sigma-k_{2\sigma})+\sum_\sigma(1-\cos(l_\sigma-k_{2\sigma}))\pm m_0\right]}{2\lambda_\pm(l-k_2)}\right)\times\right. \\ &\qquad\quad \left(\frac{\gamma_5\left[i\gamma_\lambda\sin(l_\lambda+p_\lambda)+\sum_\lambda(1-\cos(l_\lambda+p_\lambda))\pm m_0\right]}{2\lambda_\pm(l+p)}\right) \end{aligned}$$

$$
\left. \left. - \left(\frac{\gamma_5 \left[i\gamma_\rho \sin l_\rho + \sum_\rho (1-\cos l_\rho) \pm m_0 \right]}{2\lambda_\pm(l)} \right) \times \left(\frac{\gamma_5 \left[i\gamma_\sigma \sin(l_\sigma - k_{2\sigma}) + \sum_\sigma (1-\cos(l_\sigma - k_{2\sigma})) \pm m_0 \right]}{2\lambda_\pm(l-k_2)} \right) \right\} \right] . \tag{C.39}
$$

By the charge conjugation transformation, we can see that this vertex is satisfy the relation

$$
\Gamma_{1\,WZW\,\mu}(k_1,k_2) = \Gamma_{1\,WZW\,\mu}(k_2,k_1). \tag{C.40}
$$

Then, this contribution turns out to be proportional to the anomaly coefficient,

$$
\begin{aligned}
i\Delta S_{1\,WZW}[\pi^2;A] &= \sum_{rep.} \frac{1}{2} d^{abc} \sum_{n,l_1,l_2} A^a_{n\mu} \pi^b_{l_1} \pi^c_{l_2} \times \\
&\qquad \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \, e^{ik_1(l_1-m)+ik_2(l_2-m)} \, \Gamma_{1\,WZW\,\mu}(k_1,k_2),
\end{aligned} \tag{C.41}
$$

and vanishes identically for anomaly free theories. Even for anomalous case, it is easily seen by the γ_5 book-keeping that $\Gamma_{1\,WZW\,\mu}$ itself vanishes because of the vanishing trace in the spinor space.

Next we turn to the property of the next-to-leading term $i\Delta S_{2\,WZW}$. Its explicit formula is given as

$$
\begin{aligned}
i\Delta S_{2\,WZW}[\pi^2;A^2] &= \sum_{rep.} \sum_{n,m,l_1,l_2} \mathrm{Tr}\left\{ A_{n\mu} A_{m\nu} \pi_{l_1} \pi_{l_2} \right\} \times \\
&\qquad \int \frac{d^4p}{(2\pi)^4} \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \, e^{ip(m-n)+ik_1(l_1-n)+ik_2(l_2-n)} \times \\
&\qquad\qquad \Gamma_{2\,WZW\,\mu\nu}(p,k_1,k_2),
\end{aligned} \tag{C.42}
$$

where $(q+p+k_1+k_2=0)$

$$
\begin{aligned}
&\Gamma_{2\,WZW\,\mu\nu}(p,k_1,k_2) \\
&= -\frac{1}{2}\lambda^2 \int \frac{d^4l}{(2\pi)^4} \frac{2}{\lambda_\pm(l+q)+\lambda_\pm(l)} \frac{2}{\lambda_\pm(l-p)+\lambda_\pm(l)} \frac{1}{\lambda_\pm(l)} \times \\
&\quad \mathrm{Tr}\left[\gamma_5 \left[i\gamma_\mu \cos\left(l_\mu + \frac{q_\mu}{2} \right) + \sin\left(l_\mu + \frac{q_\mu}{2} \right) \right] \times \right. \\
&\quad \gamma_5 \left[i\gamma_\rho \sin l_\rho + \sum_\mu (1-\cos l_\rho) \pm m_0 \right] \times \\
&\quad \gamma_5 \left[i\gamma_\mu \cos\left(l_\nu - \frac{p_\nu}{2} \right) + \sin\left(l_\nu - \frac{p_\nu}{2} \right) \right] \times \\
&\quad \left\{ \left(\frac{\gamma_5 \left[i\gamma_\sigma \sin(l_\sigma - p_\sigma - k_{2\sigma}) + \sum_\sigma (1-\cos(l_\sigma - p_\sigma - k_{2\sigma})) \pm m_0 \right]}{2\lambda_\pm(l-p-k_2)} \right) \times \right.
\end{aligned}
$$

$$
\begin{aligned}
&\left(\frac{\gamma_5\left[i\gamma_\lambda \sin(l_\lambda+q_\lambda)+\sum_\lambda(1-\cos(l_\lambda+q_\lambda))\pm m_0\right]}{2\lambda_\pm(l+q)}\right) \\
&-\left(\frac{\gamma_5\left[i\gamma_\sigma \sin(l_\sigma-p_\sigma)+\sum_\sigma(1-\cos(l_\sigma-p_\sigma))\pm m_0\right]}{2\lambda_\pm(l-p)}\right)\times \\
&\left(\frac{\gamma_5\left[i\gamma_\lambda \sin(l_\lambda-p_\lambda-k_{2\lambda})+\sum_\lambda(1-\cos(l_\lambda-p_\lambda-k_{2\lambda}))\pm m_0\right]}{2\lambda_\pm(l-p-k_2)}\right)\Bigg\}\Bigg].
\end{aligned}
\tag{C.43}
$$

By the charge conjugation transformation again, we can see that this vertex is satisfy the relation

$$
\Gamma_{2\,WZW\,\mu}(p,k_1,k_2)=\Gamma_{2\,WZW\,\mu}(q,k_2,k_1). \tag{C.44}
$$

Then, this contribution also turns out to be proportional to the anomaly coefficient,

$$
\begin{aligned}
&i\Delta S_{2\,WZW}[\pi^2;A^2] \\
&=\sum_{rep.}\frac{1}{8}(-if^{abe}d^{cde}-id^{abe}f^{cde})\sum_{n,m,l_1,l_2}A^a_{n\mu}A^b_{m\nu}\pi^c_{l_1}\pi^c_{l_2}\times \\
&\quad\int\frac{d^4p}{(2\pi)^4}\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,e^{ip(m-n)+ik_1(l_1-n)+ik_2(l_2-n)}\times \\
&\qquad\Gamma_{2\,WZW\,\mu\nu}(p,k_1,k_2),
\end{aligned}
\tag{C.45}
$$

On the other hand, the factor of the trace of the spinor space is evaluated as

$$
\begin{aligned}
&\frac{1}{4\lambda_\pm(l-p-k_2)\lambda_\pm(l+q)}\times \\
&\Big\{+i\epsilon_{\rho\nu\sigma\lambda}\sin\left(l_\mu+\frac{q_\mu}{2}\right)\sin l_\rho\cos\left(l_\nu-\frac{p_\nu}{2}\right)\sin(l_\sigma-p_\sigma-k_{2\sigma})\sin(l_\lambda+q_\lambda) \\
&-i\epsilon_{\mu\nu\sigma\lambda}\cos\left(l_\mu+\frac{q_\mu}{2}\right)(B(l)\pm m_0)\cos\left(l_\nu-\frac{p_\nu}{2}\right)\sin(l_\sigma-p_\sigma-k_{2\sigma})\sin(l_\lambda+q_\lambda) \\
&+i\epsilon_{\mu\rho\sigma\lambda}\cos\left(l_\mu+\frac{q_\mu}{2}\right)\sin l_\rho\sin\left(l_\nu-\frac{p_\nu}{2}\right)\sin(l_\sigma-p_\sigma-k_{2\sigma})\sin(l_\lambda+q_\lambda) \\
&-i\epsilon_{\mu\rho\nu\lambda}\cos\left(l_\mu+\frac{q_\mu}{2}\right)\sin l_\rho\cos\left(l_\nu-\frac{p_\nu}{2}\right)(B(l-p-k_2)\pm m_0)\sin(l_\lambda+q_\lambda) \\
&+i\epsilon_{\mu\rho\nu\sigma}\cos\left(l_\mu+\frac{q_\mu}{2}\right)\sin l_\rho\cos\left(l_\nu-\frac{p_\nu}{2}\right)\sin(l_\sigma-p_\sigma-k_{2\sigma})(B(l+q)\pm m_0)\Big\} \\
&-\frac{1}{4\lambda_\pm(l-p)\lambda_\pm(l-p-k_2)}\times \\
&\Big\{+i\epsilon_{\rho\nu\sigma\lambda}\sin\left(l_\mu+\frac{q_\mu}{2}\right)\sin l_\rho\cos\left(l_\nu-\frac{p_\nu}{2}\right)\sin(l_\sigma-p_\sigma)\sin(l_\lambda-k_{2\lambda}-p_\lambda) \\
&-i\epsilon_{\mu\nu\sigma\lambda}\cos\left(l_\mu+\frac{q_\mu}{2}\right)(B(l)\pm m_0)\cos\left(l_\nu-\frac{p_\nu}{2}\right)\sin(l_\sigma-p_\sigma)\sin(l_\lambda-p_\lambda-k_{2\lambda}) \\
&+i\epsilon_{\mu\rho\sigma\lambda}\cos\left(l_\mu+\frac{q_\mu}{2}\right)\sin l_\rho\sin\left(l_\nu-\frac{p_\nu}{2}\right)\sin(l_\sigma-p_\sigma)\sin(l_\lambda-p_\lambda-k_{2\lambda})
\end{aligned}
$$

$$
\begin{aligned}
&-i\epsilon_{\mu\rho\nu\lambda}\cos\left(l_\mu+\frac{q_\mu}{2}\right)\sin l_\rho\cos\left(l_\nu-\frac{p_\nu}{2}\right)(B(l-p)\pm m_0)\sin(l_\lambda-p_\lambda-k_{2\lambda})\\
&+i\epsilon_{\mu\rho\nu\sigma}\cos\left(l_\mu+\frac{q_\mu}{2}\right)\sin l_\rho\cos\left(l_\nu-\frac{p_\nu}{2}\right)\sin(l_\sigma-p_\sigma)(B(l-p-k_2)\pm m_0)\Big\}.
\end{aligned}
\tag{C.46}
$$

From this expression, we can see that the trace vanishes if we set $p=0$ and $q=0(k_1+k_2=0)$. Therefore $i\Delta S_{2\,WZW}$ satisfy the following property

$$
\Gamma_{2\,WZW\,\mu\nu}(p,k_1,k_2)=0, \quad \text{at } p=0, k_1+k_2=0. \tag{C.47}
$$

In summary, $i\Delta S_{1\,WZW}$ vanishes identically for anomaly free theories as well as anomalous ones. $i\Delta S_{2\,WZW}$ vanishes identically for anomaly free theories. It vanishes at the kinematical limit $p=0$ and $q=0(k_1+k_2=0)$ for anomalous theories. In fact, the latter property of $i\Delta S_{2\,WZW}$ and the vanishing $i\Delta S_{1\,WZW}$ are sufficient to prove the one-loop renormalizability of the pure gauge model, as we will show in the next section.

D One-loop renormalizability

In the text, we have shown that the pure gauge model of the anomaly-free non-abelian chiral gauge theory defined through the vacuum overlap is renormalizable at one-loop. In this appendix, we discuss the one-loop renormalizability of the pure gauge model from a more general point of view which could cover anomalous theories, as well. We will see that even for anomalous theories, the pure gauge model is one-loop renormalizable.

It is instructive to have a general formula for the quantum correction to the classical action without taking account of the properties of $i\Delta S_{WZW}$ specific to anomaly-free theories. The quantum correction to the classical action can be expressed up to the fourth order in $A_{n\mu}$, c and $\bar{c}$ as follows:

$$
\begin{aligned}
\Delta S_1[A] &= \langle S_1[A]\rangle_0+\langle i\Delta S_{1\,WZW}[A]\rangle_0, && \text{(D.1)}\\
\Delta S_2[A^2] &= \langle S_2[A^2]\rangle_0+\frac{1}{2!}\langle S_1[A]^2\rangle_0-\frac{1}{2!}\langle \Delta S_{1\,WZW}[A]^2\rangle_0 \\
&\quad +\langle i\Delta S_{2\,WZW}[A^2]\rangle_0+\langle i\Delta S_{1\,WZW}[A]\,S_1[A]\rangle_0, && \text{(D.2)}\\
\Delta S_{0,1}[(c,\bar{c})] &= \langle S_{2c}[(c,\bar{c})]\rangle_0+\frac{1}{2}\langle S_{1c}[c/\bar{c}]^2\rangle_0, && \text{(D.3)}\\
\Delta S_{1,1}[A,(c,\bar{c})] &= \langle S_{2c}[A,(c,\bar{c})]\rangle_0+\langle S_{2c}[(c,\bar{c})]\,S_1[A]\rangle_0 \\
&\quad +\langle S_{1c}[c/\bar{c}]\,S_{1c}[A,c/\bar{c}]\rangle_0 \\
&\quad +\langle i\Delta S_{1\,WZW}[A]\,S_{2c}[(c,\bar{c})]\rangle_0 && \text{(D.4)}\\
\Delta S_3[A^3] &= \langle S_1[A^3]\rangle_0+\langle S_1[A]S_2[A^2]\rangle_0+\frac{1}{3!}\langle S_1[A]^3\rangle_0 \\
&\quad -\frac{1}{2}\langle \Delta S_{1\,WZW}[A]^2\,S_1[A]\rangle_0 \\
&\quad -\langle \Delta S_{1\,WZW}[A]\,\Delta S_{2\,WZW}[A^2]\rangle_0
\end{aligned}
$$

$$
\begin{aligned}
&+\langle i\Delta S_{3\,WZW}[A^3]\rangle_0 + \langle i\Delta S_{2\,WZW}[A^2]S_1[A]\rangle_0 \\
&+\langle i\Delta S_{1\,WZW}[A]S_2[A^2]\rangle_0 - \frac{1}{3!}\langle i\Delta S_{1\,WZW}[A]^3\rangle_0 \\
&+\frac{1}{2}\langle i\Delta S_{1\,WZW}[A]S_1[A]^2\rangle_0,
\end{aligned}
\tag{D.5}
$$

$$
\begin{aligned}
\Delta S_4[A^4] \;\; = \;\; & \langle S_2[A^4]\rangle_0 + \langle S_1[A]S_1[A^3]\rangle_0 + \frac{1}{2!}\langle S_2[A^2]^2\rangle_0 \\
&+\frac{1}{2}\langle S_1[A]^2 S_2[A^2]\rangle_0 + \frac{1}{4!}\langle S_1[A]^4\rangle_0 \\
&-\frac{1}{2!}\langle \Delta S_{2\,WZW}[A^2]^2\rangle_0 - \frac{1}{2}\langle \Delta S_{1\,WZW}[A]^2 S_2[A^2]\rangle_0 \\
&-\langle \Delta S_{3\,WZW}[A^3]\Delta S_{1\,WZW}[A]\rangle_0 \\
&-\frac{1}{2}\langle \Delta S_{1\,WZW}[A^1]^2 S_1[A]^2\rangle_0 \\
&+\frac{1}{4!}\langle \Delta S_{1\,WZW}[A^1]^4\rangle_0 \\
&+\langle i\Delta S_{4\,WZW}[A^4]\rangle_0 + \langle i\Delta S_{3\,WZW}[A^2]S_1[A]\rangle_0 \\
&+\langle i\Delta S_{2\,WZW}[A^2]S_2[A^2]\rangle_0 \\
&+\langle i\Delta S_{1\,WZW}[A]S_1[A^3]\rangle_0 + \langle i\Delta S_{1\,WZW}[A]S_1[A]S_2[A^2]\rangle_0 \\
&+\frac{1}{3!}\langle i\Delta S_{1\,WZW}[A]S_1[A]^3\rangle_0 - \frac{1}{3!}\langle i\Delta S_{1\,WZW}[A]^3 S_1[A]\rangle_0.
\end{aligned}
\tag{D.6}
$$

The local operators of dimensions less than five which can be written in terms of the pure gauge potential A_μ and the ghost and anti-ghost fields, c and $\bar{c}$, preserve parity and charge conjugation. Therefore the imaginary parts of the above expressions do not contribute to these operators. Then we can see from the above expansion that if $i\Delta S_{1\,WZW}[A]$ vanishes identically, the nontrivial contribution from the imaginary action $i\Delta\Gamma_{WZW}$ to the local operators of dimensions less than five first appear at the order $\mathcal{O}(A^4)$ in $\Delta S_4[A^4]$ as

$$
-\frac{1}{2!}\langle \Delta S_{2\,WZW}[\pi^2; A^2]^2\rangle_0. \tag{D.7}
$$

This contribution, however, turns out to vanish by the following reason. We can first take the local operator limit of the product of the two pure gauge vector potential A_μ's in each vertex: this corresponds to the kinematical limit $p=0$ of $\Gamma_{2\,WZW}(p,k_1,k_2)$ in Eq. (3.25). By the Gaussian functional integration, the two π's in one vertex are "Wick-contracted" to the two π's of the other vertex. The local operator limit of the four A_μ's

$$
(A_{n\mu}A_{n\nu})\,(A_{l\lambda}A_{l\sigma}) \longrightarrow A_{n\mu}A_{n\nu}A_{n\lambda}A_{n\sigma} + (A_{n\mu}A_{n\nu})\,(l-n)_\gamma\nabla_\gamma\,(A_{n\lambda}A_{n\sigma}) + \cdots, \tag{D.8}
$$

then corresponds to the kinematical limit of the vanishing net momentum flow through the two propagators connecting the two vertexes. This is nothing but

the kinematical limit $k_1+k_2=0$ of the vertex $\Gamma_{2\,WZW}(p=0,k_1,k_2)$, at which it vanishes. We should note that this limit is IR finite because we have the mass term for the IR regularization. The momentum dependent terms should give the local operators of dimension higher than four.

Therefore, the pure gauge model is renormalizable at one-loop, both for anomaly-free theories and for anomalous theories. This means that the quantum and dynamical effect of anomaly will show up at higher order, unlike the two-dimensional case.

Using the explicit expression, it is also not difficult to see that the imaginary contribution in Eq. (E.2) vanishes:

$$\langle i\Delta S_{2\,WZW}[\pi^2;A^2]\rangle_0=0. \tag{D.9}$$

E Detail of one-loop calculation

In this appendix, we describe in some detail the calculation of the one-loop contributions, Eqs. (3.32), (3.33), (3.34) and (3.35):

$$\begin{aligned}
\Delta S_1[A] &= \langle S_1[A]\rangle_0, && \text{(E.1)}\\
\Delta S_2[A^2] &= \langle S_2[A^2]\rangle_0+\frac{1}{2!}\langle S_1[A]^2\rangle_0, && \text{(E.2)}\\
\Delta S_{0,1}[(c,\bar{c})] &= \langle S_{2c}[(c,\bar{c})]\rangle_0+\frac{1}{2}\langle S_{1c}[c/\bar{c}]^2\rangle_0, && \text{(E.3)}\\
\Delta S_{1,1}[A,(c,\bar{c})] &= \langle S_{2c}[A,(c,\bar{c})]\rangle_0+\langle S_{2c}[(c,\bar{c})]\,S_1[A]\rangle_0\\
&\quad+\langle S_{1c}[c/\bar{c}]\,S_{1c}[A,c/\bar{c}]\rangle_0. && \text{(E.4)}
\end{aligned}$$

The propagators of the quantum fluctuations of the gauge freedom and the ghost fields, π and $\xi,\bar{\xi}$, are given by

$$\langle\pi^a_n\pi^b_m\rangle = \delta^{ab}\int\frac{d^4p}{(2\pi)^4}e^{ip(n-m)}\frac{1}{\left(\sum_\mu 4\sin^2\frac{p_\mu}{2}\right)^2+\mu^4}\equiv\delta^{ab}G(n-m), \tag{E.5}$$

$$\langle\xi^a_n\bar{\xi}^b_m\rangle = \delta^{ab}\int\frac{d^4p}{(2\pi)^4}e^{ip(n-m)}\frac{1}{\sum_\mu 4\sin^2\frac{p_\mu}{2}}\equiv\delta^{ab}G_c(n-m). \tag{E.6}$$

The contribution, Eq. (E.1), is easily seen to vanish because of the anti-symmetric nature of the group structure constant in the vertex, Eq. (A.4).

$$\langle S_1[A]\rangle_0=0. \tag{E.7}$$

The first term in Eq. (E.2) is evaluated as follows:

$$\langle S_2[A^2]\rangle_0 = \Bigg\langle\sum_n\frac{1}{2}\bar{\nabla}_\nu\hat{A}^a_{n\nu}\times$$

$$
\begin{aligned}
& \bar{\nabla}_\mu \left\{ \mathrm{Tr}\left[(\pi_{n+\hat{\mu}}[\pi_{n+\hat{\mu}}, T^a] \right.\right. \\
& \qquad +\pi_n[\pi_n, T^a] + [T^a, \pi_{n+\hat{\mu}}]\pi_{n+\hat{\mu}} \\
& \qquad \left.\left. +[T^a, \pi_n]\pi_n + 2\nabla_\mu \pi_n T^a \nabla_\mu \pi_n) \sin A_{n\mu} \right] \right\} \Big\rangle_0 \\
- & \left\langle \sum_n \frac{1}{8} f^{abc} \left(2\pi_n^b \bar{\nabla}_\nu \hat{A}^c_{n\nu} + \nabla_\nu \pi_n^b \hat{A}^c_{n\nu} + \bar{\nabla}_\nu \pi_n^b \hat{A}^c_{n-\hat{\nu},\nu} \right) \times \right. \\
& \quad \left. f^{ade} \left(2\pi_n^d \bar{\nabla}_\mu \hat{A}^e_{n\mu} + \nabla_\mu \pi_n^d \hat{A}^e_{n\mu} + \bar{\nabla}_\mu \pi_n^d \hat{A}^e_{n-\hat{\mu},\mu} \right) \right\rangle_0 \\
- & \left\langle \sum_n \nabla^2 \pi_n^a \bar{\nabla}_\mu \left\{ \mathrm{Tr}\left[\{T^a, T^b\} (\cos A_{n\mu} - 1) \right] \nabla_\mu \pi_n^b \right\} \right\rangle_0 \\
+ & \left\langle \sum_n \bar{\xi}_n^a \bar{\nabla}_\mu \left\{ \mathrm{Tr}\left[\{T^a, T^b\} (\cos A_{n\mu} - 1) \right] \nabla_\mu \xi_n^b \right\} \right\rangle_0 \\
= & \sum_n \frac{1}{2} \bar{\nabla}_\mu \hat{A}^a_{n\mu} \bar{\nabla}_\nu \hat{A}^b_{n\nu} \left\{ G(0) f^{acd} f^{bcd} \right. \\
& \left. +4\left[G(0) - G(1) \right] \left(\frac{1}{4N}\delta^{ab} - \frac{1}{8} f^{acd} f^{bcd} + \frac{1}{8} d^{acd} d^{bcd} \right) \right\} \\
- & \sum_n \frac{1}{8} \bar{\nabla}_\mu \hat{A}^a_{n\mu} \bar{\nabla}_\nu \hat{A}^b_{n\nu} \, f^{acd} f^{bcd} \left\{ 4G(0) - 4\left[G(0) - G(1) \right] \right\} \\
- & \sum_n \frac{1}{8} \hat{A}^a_{n\mu} \hat{A}^b_{n\nu} \, f^{acd} f^{bcd} \left[G(\mu-\nu) + G(0) - G(\mu) - G(-\nu) \right] \\
- & \sum_n \frac{1}{8} \hat{A}^a_{n-\hat{\mu},\mu} \hat{A}^b_{n-\hat{\nu},\nu} \, f^{acd} f^{bcd} \\
& \quad \left[G(-\mu+\nu) + G(0) - G(-\mu) - G(\nu) \right] \\
- & \sum_n \frac{1}{8} \hat{A}^a_{n\mu} \hat{A}^b_{n-\hat{\nu},\nu} \, f^{acd} f^{bcd} \left[G(\mu) + G(\nu) - G(\mu+\nu) - G(0) \right] \\
- & \sum_n \frac{1}{8} \hat{A}^a_{n-\hat{\mu},\mu} \hat{A}^b_{n\nu} \, f^{acd} f^{bcd} \times \\
& \quad \left[G(-\mu) + G(-\nu) - G(-\mu-\nu) - G(0) \right] \\
- & \sum_n (\nabla^2)^2 G(0) \left(\frac{N^2-1}{N} \right) \mathrm{Tr}\left(\cos A_{n\mu} - 1 \right) \\
- & \sum_n (\nabla^2) G_c(0) \left(\frac{N^2-1}{N} \right) \mathrm{Tr}\left(\cos A_{n\mu} - 1 \right)
\end{aligned}
\tag{E.8}
$$

Noting the relations, $f^{acd}f^{bcd} = N\delta^{ab}$ and $d^{acd}d^{bcd} = \frac{N^2-4}{N}\delta^{ab}$ and using the identities

$$
(\nabla^2)^2 G(n-m) \quad = \quad \delta_{nm} - \mu^4 G(n-m), \tag{E.9}
$$

$$\nabla^2 G_c(n-m) \quad = \quad -\delta_{nm}, \tag{E.10}$$

the above expression is reduced to

$$\begin{aligned}
\langle S_2[A^2]\rangle_0 \quad &= \quad \sum_n \left(\frac{N^2-1}{2N}\right) [G(0)-G(1)]\ \bar{\nabla}_\mu \hat{A}^a_{n\mu} \bar{\nabla}_\nu \hat{A}^a_{n\nu} \\
&- \quad \sum_n \frac{N}{8} \hat{A}^a_{n\mu} \hat{A}^a_{n\nu}\ [G(\mu-\nu)+G(0)-G(\mu)-G(-\nu)] \\
&- \quad \sum_n \frac{N}{8} \hat{A}^a_{n-\hat{\mu},\mu} \hat{A}^a_{n-\hat{\nu},\nu}\ [G(-\mu+\nu)+G(0)-G(-\mu)-G(\nu)] \\
&- \quad \sum_n \frac{N}{8} \hat{A}^a_{n\mu} \hat{A}^a_{n-\hat{\nu},\nu}\ [G(\mu)+G(\nu)-G(\mu+\nu)-G(0)] \\
&- \quad \sum_n \frac{N}{8} \hat{A}^a_{n-\hat{\mu},\mu} \hat{A}^a_{n\nu}\ [G(-\mu)+G(-\nu)-G(-\mu-\nu)-G(0)] \\
&+ \quad \mathcal{O}(\mu^4).
\end{aligned} \tag{E.11}$$

The explicit formula of the second term in Eq. (E.2) is given as follows:

$$\begin{aligned}
\frac{1}{2!}\langle S_1[A]^2\rangle_0 \quad = \quad \frac{1}{2!}\Bigg\langle \Bigg(&\sum_{n\mu} \frac{1}{2} \bar{\nabla}_\mu \hat{A}^a_{n\mu} f^{abc} \pi^b_n \nabla^2 \pi^c_n \\
&- \sum_{n\mu} \frac{1}{2} \nabla^2 \pi^a_n f^{abc} \left(\nabla_\mu \pi^b_n \hat{A}^c_{n\mu} + \bar{\nabla}_\mu \pi^b_n \hat{A}^c_{n-\hat{\mu},\mu} \right) \\
&+ \sum_n \frac{1}{2} f^{abc} \bar{\xi}^a_n \bar{\nabla}_\mu \left\{ \left(\xi^b_n + \xi^b_{n+\hat{\mu}} \right) \hat{A}^c_{n\mu} \right\} \Bigg)^2 \Bigg\rangle_0
\end{aligned} \tag{E.12}$$

It can be divided into the four contributions and they are evaluated separately as follows:

$$\begin{aligned}
&\frac{1}{2!}\left\langle \left(\sum_{n\mu} \frac{1}{2} \bar{\nabla}_\mu \hat{A}^a_{n\mu} f^{abc} \pi^b_n \nabla^2 \pi^c_n \right)^2 \right\rangle_0 \\
&\qquad = \frac{N}{8} \sum_{nm} \bar{\nabla}_\mu \hat{A}^a_{n\mu} \bar{\nabla}_\nu \hat{A}^a_{m\nu} \times \\
&\qquad\quad \left[G(n-m)(\nabla^2)^2 G(m-n) - \nabla^2 G(n-m) \nabla^2 G(m-n) \right],
\end{aligned} \tag{E.13}$$

$$\begin{aligned}
&\frac{1}{2!}\left\langle \left(-\sum_{n\mu} \frac{1}{2} \nabla^2 \pi^a_n f^{abc} \left(\nabla_\mu \pi^b_n \hat{A}^c_{n\mu} + \bar{\nabla}_\mu \pi^b_n \hat{A}^c_{n-\hat{\mu},\mu} \right) \right)^2 \right\rangle_0 \\
&= \frac{N}{8} \sum_{nm} \hat{A}^a_{n\mu} \hat{A}^a_{m\nu}\ (\nabla^2)^2 G(n-m) \times
\end{aligned}$$

$$[G(m+\hat{\nu}-n-\hat{\mu})+G(m-n)-G(m+\hat{\nu}-n)-G(m-n-\hat{\mu})]$$

$$+\frac{N}{8}\sum_{nm}\hat{A}^a_{n-\hat{\mu},\mu}\hat{A}^a_{m-\hat{\nu},\nu}\ (\nabla^2)^2G(n-m)\times$$

$$[G(m-n)+G(m-\hat{\nu}-n+\hat{\mu})-G(m-\hat{\nu}-n)-G(m-n+\hat{\mu})]$$

$$+\frac{N}{8}\sum_{nm}\hat{A}^a_{n-\hat{\mu},\mu}\hat{A}^a_{m\nu}\ (\nabla^2)^2G(n-m)\times$$

$$[G(m+\hat{\nu}-n)+G(m-n+\hat{\mu})-G(m-n)-G(m+\hat{\nu}-n+\hat{\mu})]$$

$$+\frac{N}{8}\sum_{nm}\hat{A}^a_{n\mu}\hat{A}^a_{m-\hat{\nu},\nu}\ (\nabla^2)^2G(n-m)\times$$

$$[G(m-n-\hat{\mu})+G(m-\hat{\nu}-n)-G(m-\hat{\nu}-n-\hat{\mu})-G(m-n)]$$

$$-\frac{N}{8}\sum_{nm}\hat{A}^a_{n\mu}\hat{A}^a_{m\nu}\ \times$$

$$\left[\nabla^2G(n-m-\hat{\nu})-\nabla^2G(n-m)\right]\ \left[\nabla^2G(m-n-\hat{\mu})-\nabla^2G(m-n)\right]$$

$$-\frac{N}{8}\sum_{nm}\hat{A}^a_{n-\hat{\mu},\mu}\hat{A}^a_{m-\hat{\nu},\nu}\ \times$$

$$\left[\nabla^2G(n-m)-\nabla^2G(n-m+\hat{\nu})\right]\ \left[\nabla^2G(m-n)-\nabla^2G(m-n+\hat{\mu})\right]$$

$$-\frac{N}{8}\sum_{nm}\hat{A}^a_{n-\hat{\mu},\mu}\hat{A}^a_{m\nu}\ \times$$

$$\left[\nabla^2G(n-m-\hat{\nu})-\nabla^2G(n-m)\right]\ \left[\nabla^2G(m-n)-\nabla^2G(m-n+\hat{\mu})\right]$$

$$-\frac{N}{8}\sum_{nm}\hat{A}^a_{n\mu}\hat{A}^a_{m-\hat{\nu},\nu}\ \times$$

$$\left[\nabla^2G(n-m)-\nabla^2G(n-m+\hat{\nu})\right]\ \left[\nabla^2G(m-n-\hat{\mu})-\nabla^2G(m-n)\right], \tag{E.14}$$

$$\left\langle\left(\sum_{n\mu}\frac{1}{2}\bar{\nabla}_\mu\hat{A}^a_{n\mu}f^{abc}\pi^b_n\nabla^2\pi^c_n\right)\times\right.$$

$$\left.\left(-\sum_{n\mu}\frac{1}{2}\nabla^2\pi^a_nf^{abc}\left(\nabla_\mu\pi^b_n\hat{A}^c_{n\mu}+\bar{\nabla}_\mu\pi^b_n\hat{A}^c_{n-\hat{\mu},\mu}\right)\right)\right\rangle_0$$

$$=-\frac{N}{4}\sum_{nm}\bar{\nabla}_\mu\hat{A}^a_{n\mu}\hat{A}^a_{m\nu}\ \times$$

$$\nabla^2G(n-m)\left[\nabla^2G(m+\hat{\nu}-n)-\nabla^2G(m-n)\right]$$

$$-\frac{N}{4}\sum_{nm}\bar{\nabla}_\mu\hat{A}^a_{n\mu}\hat{A}^a_{m-\hat{\nu},\nu}\ \times$$

$$\nabla^2G(n-m)\left[\nabla^2G(m-n)-\nabla^2G(m-\hat{\nu}-n)\right]$$

$$+\frac{N}{4}\sum_{nm}\bar{\nabla}_\mu\hat{A}^a_{n\mu}\hat{A}^a_{m\nu}\ \times$$

$$
\begin{aligned}
&\quad [G(n-m-\hat{\nu}) - G(n-m)]\,(\nabla^2)^2 G(m-n) \\
&+\frac{N}{4}\sum_{nm} \bar{\nabla}_\mu \hat{A}^a_{n\mu}\hat{A}^a_{m-\hat{\nu},\nu}\ \times \\
&\quad [G(n-m) - G(n-m+\hat{\nu})]\,(\nabla^2)^2 G(m-n),
\end{aligned}
\tag{E.15}
$$

$$
\begin{aligned}
&\frac{1}{2!}\left\langle\left(\sum_n \frac{1}{2} f^{abc}\bar{\xi}^a_n \bar{\nabla}_\mu \left\{\left(\xi^b_n + \xi^b_{n+\hat{\mu}}\right)\hat{A}^c_{n\mu}\right\}\right)^2\right\rangle_0 \\
&= \frac{N}{8}\sum_{nm} \hat{A}^a_{n\mu}\hat{A}^a_{m,\nu}\ \times \\
&\quad [G_c(n-m-\hat{\nu}) - G_c(n-m)]\ [G_c(m-n-\hat{\mu}) - G_c(m-n)] \\
&+\frac{N}{8}\sum_{nm} \hat{A}^a_{n\mu}\hat{A}^a_{m,\nu}\ \times \\
&\quad [G_c(n+\hat{\mu}-m-\hat{\nu}) - G_c(n+\hat{\mu}-m)] \times \\
&\qquad [G_c(m+\hat{\nu}-n-\hat{\mu}) - G_c(m+\hat{\nu}-n)] \\
&+\frac{N}{8}\sum_{nm} \hat{A}^a_{n\mu}\hat{A}^a_{m,\nu}\ \times \\
&\quad [G_c(n-m-\hat{\nu}) - G_c(n-m)]\ [G_c(m+\hat{\nu}-n-\hat{\mu}) - G_c(m+\hat{\nu}-n)] \\
&+\frac{N}{8}\sum_{nm} \hat{A}^a_{n\mu}\hat{A}^a_{m,\nu}\ \times \\
&\quad [G_c(n+\hat{\mu}-m-\hat{\nu}) - G_c(n+\hat{\mu}-m)]\ [G_c(m-n-\hat{\mu}) - G_c(m-n)]\,.
\end{aligned}
\tag{E.16}
$$

As for the second contribution, we can see by noting the identity,

$$
\nabla^2 G(n-m) = -G_c(n-m) + \mathcal{O}(\mu^4), \tag{E.17}
$$

that the last four terms are canceled by the forth contribution of the ghost fields.

Under the assumption that the classical solution is sufficiently slowly varying,

$$
\nabla_\nu A_{n\mu} << A_{n\mu}, \tag{E.18}
$$

we extract the first order term in the local operator limit,

$$
A^a_{m\nu} \longrightarrow A^a_{n\nu} + (m-n)\bar{\nabla}_\nu A^a_{n\nu} + \cdots. \tag{E.19}
$$

Then the first contribution turns out to vanishes. In the second contribution, the remaining four terms are already local and turn out to be

$$
\begin{aligned}
&\frac{N}{8}\sum_{n} \hat{A}^a_{n\mu}\hat{A}^a_{n\nu}\ [G(\hat{\nu}-\hat{\mu}) + G(0) - G(\hat{\nu}) - G(-\hat{\mu})] \\
&+\frac{N}{8}\sum_{n} \hat{A}^a_{n-\hat{\mu},\mu}\hat{A}^a_{n-\hat{\nu},\nu}\ [G(0) + G(-\hat{\nu}+\hat{\mu}) - G(-\hat{\nu}) - G(\hat{\mu})]
\end{aligned}
$$

$$+\frac{N}{8}\sum_{n}\hat{A}^a_{n-\hat{\mu},\mu}\hat{A}^a_{n\nu}\;[G(\hat{\nu})+G(\hat{\mu})-G(0)-G(\hat{\nu}+\hat{\mu})]$$
$$+\frac{N}{8}\sum_{nm}\hat{A}^a_{n\mu}\hat{A}^a_{n-\hat{\nu},\nu}\;[G(-\hat{\mu})+G(-\hat{\nu})-G(-\hat{\nu}-\hat{\mu})-G(0)]\,. \tag{E.20}$$

And we can see that they cancel the last four terms in Eq. (E.11). As to the third contribution, the nonlocal product of the correlation functions has the following local expansion:

$$\begin{aligned}&\nabla^2 G(n-m)\left[\nabla^2 G(m+\hat{\nu}-n)-\nabla^2 G(m-n)\right]\\ &\quad=\delta_{nm}\,[G(1)-G(0)]-\nabla_\nu\delta_{nm}\,\bar{G}\cdots,\end{aligned} \tag{E.21}$$

where

$$\begin{aligned}\bar{G} &= \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\frac{\left(\sum_\mu 4\sin^2\frac{k_\mu}{2}\right)\left(\sum_\mu \sin^2 k_\mu\right)}{\left[\left(\sum_\mu 4\sin^2\frac{k_\mu}{2}\right)^2+\mu_0^4\right]^2}\\ &\simeq -\frac{1}{16\pi^2}\,\ln(a\mu_0)+C.\end{aligned} \tag{E.22}$$

This is because in momentum space, the product of the correlation functions has the expression

$$\begin{aligned}&\nabla^2 G(n-m)\left[\nabla^2 G(m+\hat{\nu}-n)-\nabla^2 G(m-n)\right]\\ &\quad\equiv\int\frac{d^4p}{(2\pi)^4}e^{ip(n-m)}\,\Gamma_\nu(p),\end{aligned} \tag{E.23}$$

where

$$\Gamma_\nu(p) = \int\frac{d^4k}{(2\pi)^4}\,\frac{\left(\sum_\mu 4\sin^2\frac{(k+p)_\mu}{2}\right)}{\left(\sum_\mu 4\sin^2\frac{(k+p)_\mu}{2}\right)^2+\mu^4}\,\frac{\left(\sum_\mu 4\sin^2\frac{k_\mu}{2}\right)\left(e^{ik_\nu}-1\right)}{\left(\sum_\mu 4\sin^2\frac{k_\mu}{2}\right)^2+\mu^4}, \tag{E.24}$$

and $\Gamma_\nu(p)$ can be expanded in terms of the momentum p as

$$\Gamma_\nu(p)=[G(1)-G(0)]-ip_\nu\,\bar{G}+\mathcal{O}(\mu^4,p^2). \tag{E.25}$$

Then the local operator limit of the third contribution can be evaluated as

$$\frac{1}{2}\sum_{n}\bar{\nabla}_\mu\hat{A}^a_{n\mu}\bar{\nabla}_\nu\hat{A}^a_{n\nu}\,N\bar{G}. \tag{E.26}$$

Combining these results, we have

$$\Delta S_2[A^2] \quad = \quad \frac{1}{2}\sum_n \bar{\nabla}_\mu \hat{A}^a_{n\mu}\bar{\nabla}_\nu \hat{A}^a_{n\nu}\left(N\bar{G}+\left(\frac{N^2-1}{N}\right)[G(0)-G(1)]\right). \tag{E.27}$$

The contributions of ghost fields Eqs. (E.3) and (E.4) can be calculated in a similar manner and we finally obtain Eq. (3.40)

$$\Delta S_1+\Delta S_2+\Delta S_{1c}+\Delta S_{2c} \quad = \quad -\lambda^2\left[N\bar{G}+\left(\frac{N^2-1}{N}\right)[G(0)-G(1)]\right]\mathcal{O}_0. \tag{E.28}$$

F Effect of Gauge symmetry breaking term

In this appendix, we describe in some detail the calculation of the one-loop contributions in the case with the explicit gauge symmetry breaking term. We first give the explicit formula of the explicit gauge symmetry breaking term expanded in terms of the fluctuation of the gauge freedom, Eq. (4.3):

$$\begin{aligned} S_B[\exp(i\lambda\pi)g] \quad &= \quad S_B[g]+S_{B\,0}[\pi^2] \\ &+ \quad S_{B\,1}[\pi^2,\sin A]+S_{B\,2}[\pi^2,\cos A-1]+\mathcal{O}(\pi^3). \end{aligned} \tag{F.1}$$

$$S_{B\,0}[\pi^2] \quad = \quad -\frac{1}{2}K\lambda^2\sum_{n\mu}\nabla_\mu\pi^a_n\nabla_\mu\pi^a_n, \tag{F.2}$$

$$S_{B\,1}[\pi^2,\sin A] \quad = \quad +\frac{1}{2}K\lambda^2 f^{abc}\pi^a_n\pi^b_{n+\hat{\mu}}\hat{A}^c_{n\mu}, \tag{F.3}$$

and

$$\begin{aligned} &S_{B\,2}[\pi^2,\cos A-1] \\ &= -K\lambda^2\mathrm{Tr}\left\{\sum_\mu\left(\pi^2_n+\pi^2_{n+\hat{\mu}}-\{\pi_n,\pi_{n+\hat{\mu}}\}\right)(\cos A_{n\mu}-1)\right\}. \end{aligned} \tag{F.4}$$

The propagator of the quantum fluctuation of the gauge freedom is given by

$$\begin{aligned} \langle\pi^a_n\pi^b_m\rangle \quad &= \quad \delta^{ab}\int\frac{d^4p}{(2\pi)^4}e^{ip(n-m)}\frac{1}{\left(\sum_\mu 4\sin^2\frac{p_\mu}{2}\right)^2+M_0^2\left(\sum_\mu 4\sin^2\frac{p_\mu}{2}\right)} \\ &\equiv \quad \delta^{ab}G_B(n-m). \end{aligned} \tag{F.5}$$

Here we have set

$$M_0^2 = K\lambda^2. \tag{F.6}$$

The quantum correction to the classical action can be divided into two classes: the first class consists of the contributions which does not have the gauge-breaking vertexes from S_B and are given by the same expressions as Eqs. (3.32), (3.33), (3.34), (3.35). Corresponding equations are in the appendix E as Eqs. (E.1), (E.2), (E.3), (E.4). Here we have taken account of the fact that M_0^2 should be counted to have mass dimension two and omit Eqs. (3.36) and (3.37).

The second class consists of the additional contribution due to the gauge-breaking vertexes. They are given up to the second order in A_μ as follows:

$$\Delta S_{B\,1}[A] \quad = \quad \langle S_{B\,1}[A] \rangle_0, \tag{F.7}$$

$$\Delta S_{B\,2}[A^2] \quad = \quad \langle S_{B\,2}[A^2] \rangle_0 + \frac{1}{2} \langle S_{B\,1}[A]^2 \rangle_0 + \langle S_{B\,1}[A] S_1[A] \rangle_0. \tag{F.8}$$

The contributions of the first class can be evaluated in a similar manner described in the appendix E, by just noting the differences in the relations

$$(\nabla^2)^2 G_B(n-m) \quad = \quad \delta_{nm} + M_0^2 \nabla^2 G_B(n-m), \tag{F.9}$$

$$\nabla^2 G_B(n-m) \quad = \quad -G_c(n-m) + M_0^2 G_B(n-m), \tag{F.10}$$

and by counting M_0^2 as mass dimension two. They are given as

$$\begin{aligned}
&\langle S_2[A^2] \rangle_0 + \frac{1}{2!} \langle S_1[A]^2 \rangle_0 \\
&= \frac{1}{2} \sum_n \bar{\nabla}_\mu \hat{A}^a_{n\mu} \bar{\nabla}_\nu \hat{A}^a_{n\nu} \left(N \bar{G}_B + \left(\frac{N^2-1}{N} \right) [G_B(0) - G_B(1)] \right) \\
&\quad - M_0^2 \nabla^2 G_B(0) \left(\frac{N^2-1}{N} \right) \sum_n \mathrm{Tr} \left(\cos A_{n\mu} - 1 \right) \\
&\quad + \frac{N}{8} M_0^2 \sum_{nm} \hat{A}^a_{n\mu} \hat{A}^a_{m\nu} \;\times \\
&\qquad \big[\nabla^2 G_B(n-m) + \nabla^2 G_B(n+\hat{\mu}-m-\hat{\nu}) \\
&\qquad\qquad + \nabla^2 G_B(n+\hat{\mu}-m) + \nabla^2 G_B(n-m-\hat{\nu}) \big] \;\times \\
&\qquad\quad [G_B(m+\hat{\nu}-n-\hat{\mu}) + G_B(m-n) \\
&\qquad\qquad - G_B(m+\hat{\nu}-n) - G_B(m-n-\hat{\mu})] \\
&\quad - \frac{N}{8} \sum_{nm} \hat{A}^a_{n\mu} \hat{A}^a_{m\nu} \;\times \\
&\qquad \big[\nabla^2 G_B(n-m) - \nabla^2 G_B(n+\hat{\mu}-m-\hat{\nu}) \\
&\qquad\qquad + \nabla^2 G_B(n+\hat{\mu}-m) - \nabla^2 G_B(n-m-\hat{\nu}) \big] \;\times \\
&\qquad \big[\nabla^2 G_B(m-n) - \nabla^2 G_B(m+\hat{\nu}-n-\hat{\mu}) \\
&\qquad\qquad - \nabla^2 G_B(m-n-\hat{\mu}) + \nabla^2 G_B(m+\hat{\nu}-n) \big] \\
&\quad + \frac{N}{8} \sum_{nm} \hat{A}^a_{n\mu} \hat{A}^a_{m\nu} \;\times
\end{aligned}$$

$$
\begin{aligned}
&[G_c(n-m)+G_c(n+\hat{\mu}-m-\hat{\nu})\\
&\qquad -G_c(n+\hat{\mu}-m)-G_c(n-m-\hat{\nu})]\times\\
&[G_c(m-n)+G_c(m+\hat{\nu}-n-\hat{\mu})\\
&\qquad -G_c(m-n-\hat{\mu})-G_c(m+\hat{\nu}-n)]\,,
\end{aligned}
\tag{F.11}
$$

where

$$
\begin{aligned}
\bar{G}_B &= \frac{1}{2}\int\frac{d^4k}{(2\pi)^4}\frac{\left(\sum_\mu \sin^2 k_\mu\right)}{\left[\sum_\mu 4\sin^2\frac{k_\mu}{2}+M_0^2\right]^3}\\
&\simeq -\frac{1}{16\pi^2}\,\ln(M_0)+\bar{C}_B \qquad (M_0<<1).
\end{aligned}
\tag{F.12}
$$

The last three terms in the above are evaluated further as

$$
\begin{aligned}
&-\frac{N}{8}M_0^2\sum_{nm}\hat{A}^a_{n\mu}\hat{A}^a_{m\nu}\ \times\\
&\quad\{G_c(n-m)\ \times\\
&\qquad[-G_B(m-n)+3G_B(m+\hat{\nu}-n-\hat{\mu})\\
&\qquad -G_B(m+\hat{\nu}-n)-G_B(m-n-\hat{\mu})]\\
&\quad +G_c(n+\hat{\mu}-m-\hat{\nu})\times\\
&\qquad[3G_B(m-n)-G_B(m+\hat{\nu}-n-\hat{\mu})\\
&\qquad -G_B(m+\hat{\nu}-n)-G_B(m-n-\hat{\mu})]\\
&\quad -G_c(n+\hat{\mu}-m)\times\\
&\qquad[-G_B(m-n)-G_B(m+\hat{\nu}-n-\hat{\mu})\\
&\qquad +3G_B(m+\hat{\nu}-n)-G_B(m-n-\hat{\mu})]\\
&\quad -G_c(n-m-\hat{\nu})\times\\
&\qquad[-G_B(m-n)-G_B(m+\hat{\nu}-n-\hat{\mu})\\
&\qquad -G_B(m+\hat{\nu}-n)+3G_B(m-n-\hat{\mu})]\}+\mathcal{O}(M_0^4)\\
=\ &-\frac{N}{8}M_0^2\sum_{nm}\hat{A}^a_{n\mu}\hat{A}^a_{m\nu}\ \times\\
&\quad[\,4G_c(n-m)\,G_B(m+\hat{\nu}-n-\hat{\mu})+4G_c(n+\hat{\mu}-m-\hat{\nu})\,G_B(m-n)\\
&\quad -4G_c(n+\hat{\mu}-m)\,G_B(m+\hat{\nu}-n)-4G_c(n-m-\hat{\nu})\,G_B(m-n-\hat{\mu})\\
&\qquad -(G_c(n-m)+G_c(n+\hat{\mu}-m-\hat{\nu})\\
&\qquad\qquad -G_c(n+\hat{\mu}-m)-G_c(n-m-\hat{\nu}))\times\\
&\qquad\quad (G_B(m-n)+G_B(m+\hat{\nu}-n-\hat{\mu})\\
&\qquad\qquad +G_B(m-n-\hat{\mu})+G_B(m+\hat{\nu}-n))]+\mathcal{O}(M_0^4)\\
=\ &-N\tilde{G}_B\,M_0^2\ \frac{1}{2}\sum_n\hat{A}^a_{n\mu}\hat{A}^a_{n\nu}\ +\mathcal{O}(M_0^4),
\end{aligned}
\tag{F.13}
$$

where

$$\begin{aligned}
\tilde{G}_B &= \frac{3}{4}\int \frac{d^4k}{(2\pi)^4} \frac{\left(\sum_\mu \sin^2 k_\mu\right)}{\left[\left(\sum_\mu 4\sin^2 \frac{k_\mu}{2}\right)^2 \left(\sum_\mu 4\sin^2 \frac{k_\mu}{2} + M_0^2\right)\right]} \\
&\simeq -\frac{3}{32\pi^2}\ \ln(M_0) + \tilde{C}_B \qquad (M_0 << 1).
\end{aligned} \tag{F.14}$$

As to the contributions of the second class, it is easily seen that Eq. (F.7) vanishes. In Eq. (F.8), only the first term contributes to the local operators of dimension less than five. It is finite and is evaluated as follows:

$$\begin{aligned}
&\Delta S_{B\,2}[A^2] \\
&= M_0^2 \left(\frac{N^2-1}{2N}\right) [G_B(0) - G_B(1)] \left(\sum_{n\mu} \frac{1}{2}\, A^a_{n\mu} A^a_{n\mu}\right).
\end{aligned} \tag{F.15}$$

Combining these result, we obtain

$$\begin{aligned}
&\Delta S_1 + \Delta S_2 + \Delta S_{1c} + \Delta S_{2c} \\
&\quad = -\lambda^2 \left[N\bar{G}_B + \left(\frac{N^2-1}{N}\right)[G_B(0) - G_B(1)]\right] \mathcal{O}_0 \\
&\qquad\quad + M_0^2 \left[N\tilde{G}_B - \left(\frac{N^2-1}{2N}\right) \nabla^2 G_B(0)\right] \mathcal{O}_1, \\
&\Delta S_{B\,1} + \Delta S_{B\,2} \\
&\quad = -M_0^2 \left(\frac{N^2-1}{2N}\right) [G_B(0) - G_B(1)]\ \mathcal{O}_1.
\end{aligned} \tag{F.16}$$

References

[1] J. Smit, Acta Physica Polonica **B17** (1986) 531; P.D.V. Swift, Phys. Lett. **145B** (1984) 256.

[2] S. Aoki, Phys. Rev. Lett. **60** (1988) 2109; Phys. Rev. **D38** (1988) 618.

[3] K. Funakubo and T. Kashiwa, **60** (1988) 2113.

[4] M.F.L. Golterman, Nucl. Phys. **B**(Proc. Suppl.)**20** (1991) and and references there in; M.F.L. Golterman and D.N. Petcher, Nucl. Phys. **B**(Proc. Suppl.)**29C** (1992) 60 .

[5] E. Eichten and J. Preskill, Nucl. Phys. **B268** (1986) 179.

[6] M.F.L. Golterman, D.N. Petcher and E. Rivas, Nucl. Phys. B395 (1993) 596.

[7] J. Smit, Nucl. Phys. **B**(Proc. Suppl.)**4** (1988) 451; Nucl. Phys. **B**(Proc. Suppl.)**26** (1992) 480; Nucl. Phys. **B**(Proc. Suppl.)**29C** (1992) 83;

[8] W. Bock, J. Smit and J.C. Vink, Nucl. Phys. B414 (1994) 73; W. Bock, J. Smit and J.C. Vink, Nucl. Phys. B416 (1994) 645.

[9] D.B. Kaplan, Phys. Lett. **B288** (1992) 342.

[10] K. Jansen, Phys. Lett. B288 (1992) 348;Phys. Rept. 273 (1996) 1; K. Jansen and M. Schmaltz, Phys. Lett. B296 (1992) 374; M. Golterman, K. Jansen and D. Kaplan, Phys. Lett. B301 (1993) 219; C.P. Korthals-Altes, S. Nicolis and J. Prades, Phys. Lett. **B316** (1993) 339; A. Hulsebos, C.P. Korthals-Altes and S. Nicolis, Nucl. Phys. **B450** (1995) 437; S. Aoki, and H. Hirose, Phys. Rev. **D49** (1994) 3517; S. Chandrasekharan, Phys. Rev. D49 (1994) 1980; H. Aoki, S. Iso, J. Nishimura and M. Oshikawa, Mod. Phys. Lett. **A9** (1994) 1755; J. Distler and S. Rey, Princeton preprint, `hep-lat/9305026`; S. Aoki, and H. Hirose, Phys.Rev. **D54** (1996) 3471; S. Aoki and K. Nagai, Phys. Rev. **D53**(1996) 5058; Nucl.Phys. Proc.Suppl. **53** (1997) 635-637; UTHEP-349 (1996) `hep-lat/9610033`.

[11] D.B. Kaplan, Nucl. Phys. **B30**(Proc. Suppl.) (1993) 597.

[12] M. Golterman, K. Jansen, D. Petcher and J. Vink, Phys. Rev. **D49** (1994) 1606; M.F.L. Golterman and Y. Shamir, Phys. Rev. **D51** (1995) 3026.

[13] M. Creutz and I. Horvath, Phys. Rev. **D50** (1994) 2297.

[14] M. Creutz, M. Tytgat, C. Rebbi, S.-S. Xue, `hep-lat/9612017`.

[15] S. Aoki, K. Nagai and S.V. Zenkin, UTHEP-360, UT-CCP-P21 (1997), `hep-lat/9705001`.

[16] R. Narayanan and H. Neuberger, Nucl. Phys. **B412** (1994) 574; Phys. Rev. Lett. **71** (1993) 3251; Nucl. Phys. **B**(Proc. Suppl.)**34** (1994) 95,587; Nucl. Phys. **B443** (1995) 305.

[17] M.F.L. Golterman and Y. Shamir, Phys. Lett. **B353** (1995) 84; Erratum-ibid. **B359** (1995) 422. R. Narayanan and H. Neuberger, Phys. Lett. **B358** (1995) 303.

[18] D. Foerster, H.B. Nielsen and M. Ninomiya, Phys. Lett. B94 (1980) 135.

[19] Y. Kikukawa, KUNS-1445 HE(TH)97/08, May 1997, `hep-lat/9705024`.

[20] H. Hata, Phys. Lett. **143B** (1984) 171.

[21] Y. Kikukawa, in preparation.

[22] A. Borrelli, L. Maiani, G.C. Rossi, R. Sisto and M. Testa, Nucl. Phys. **B333** (1990) 335.

[23] H. Neuberger, Phys. Lett. B183 (1987) 337.

[24] Y. Shamir, TAUP-2306-95, Dec 1995; Y. Shamir, Nucl. Phys. **B**(Proc. Suppl.)**53** (1997) 664.

[25] M.F.L. Golterman and Y. Shamir, TAUP-2361-96, August 1996, `hep-lat/9608116`.

[26] Y. Kikukawa and S. Miyazaki, Prog. Theor. Phys. **96** (1996) 1189.

[27] L. Giusti, `hep-lat/9605032`.

[28] D. Zwanziger, Phys. Lett. **B139** (1990) 461.

[29] H. Kawai, R. Nakayama and K. Seo, Nucl. Phys. **B189** (1981) 40; M. Lüscher, Selected Topics in Lattice Field Theory, in *Fields, strings and critical phenomena*, E. Brézin and J. Zinn-Justin, eds., Les Houches session XLV (North-Holland, Amsterdam).

[30] T. Aoyama and Y. Kikukawa, Nucl. Phys. **B**(Proc. Suppl.)**53** (1997) 638.

[31] R. Dashen and D.J. Gross, Phys. Rev. **D23** (1981) 2340; J. Shigemitsu and J.B. Kogut, Nucl. Phys. **B190[FS3]** (1981) 365.

[32] B. Sharp, J. Math. Phys. **25** (1984) 3324; Ph. de Focrand and J. Hetrick, Nucl. Phys. **B**(Proc. Suppl.)**42** (1995) 862;

[33] Y. Shamir, Nucl. Phys. **B**(Proc. Suppl.)**47** (1996) 212.

[34] S. Randjbar-Daemi and J. Strathdee, Phys. Lett. **B348** (1995) 543; Nucl. Phys. **B443** (1995) 386; Nucl. Phys. **B461** (1996) 305; Nucl. Phys. **B466** (1996) 335; Phys. Lett. **B402** (1997) 134.

[35] A. Yamada, `hep-lat/9705040`.

[36] S. Elitzur, Nucl. Phys. **B212** (1983) 501; F. David, Phys. Lett. 96B (1980) 371.

[37] T. Kugo, *Quantum Theory of Gauge Fields, Vol II* Sec. 7-5, 1989, (Baifukan, Japan) (in Japanese).

[38] 't Hooft, Nucl. Phys. **B61** (1973) 455; S. Weinberg, Phys. Rev. **D8** (1973) 3497; C.G. Callan, "The New-Improved Renormalization Group for Scalar Fields", Princeton University preprint, (1973); T. Kugo, Prog. Theor. Phys. **57** (1977) 593.

[39] S. Elitzur, Phys. Rev. **D12** (1975) 3978.

[40] H. Hata and Y. Taniguchi, Prog.Theor.Phys. 94 (1995) 435-444 .

[41] H.B. Nielsen and M. Ninomiya, Nucl. Phys. **B185** (1981) 20; *Errata* Nucl. Phys. **B195** (1982) 541; Nucl. Phys. **B193** (1981) 173.

[42] Y. Shamir, Phys. Rev. Lett. **77** (1993) 2691; Nucl. Phys. **B**(Proc. Suppl.)**34** (1988) 590; `hep-lat/9307002`.

[43] C. Rebbi, Phys. Lett. **B186** (1987) 200.

[44] M. Campostrini, G. Curci and A. Pelisseto, Phys. Lett. **B193** (1987) 279; G.T. Bodwin and E.V. Kovacs, Phys. Lett. **B193** (1987) 283.

[45] S. Aoki and R.B. Levien, Phys. Rev. **D51** (1995) 3790.

[46] Y. Kikukawa, Nucl. Phys. **B**(Proc. Suppl.)**47** (1996) 599.

[47] S.A. Frolov and A.A. Slavnov, Phys. Lett. **B309** (1993) 344.

[48] M.F.L. Golterman, D.N. Petcher and J. Smit, Nucl. Phys. **B370** (1992) 51.

[49] W. Bock, A.K. De, E. Focht and J. Smit, Nucl. Phys. B401 (1993) 481.

[50] R. Narayanan, H. Neuberger and P. Vranas, Phys. Lett. **B353** (1995) 507; Nucl. Phys. **B**(Proc. Suppl.)**47** (1995) 596;
R. Narayanan and H. Neuberger, Nucl. Phys. **B477** (1996) 521; Nucl. Phys. **B**(Proc. Suppl.)**53** (1997) 661. R. Narayanan and P. Vranas, UW/PT-97-04, CU-TP-811, `hep-lat/9702005`; R. Narayanan and H. Neuberger, Phys. Lett. **B393** (1997) 360; Y. Kikukawa, R. Narayanan and H. Neuberger, Phys. Lett. **B399** (1997) 105; RU-97-28, UW/PT-97-10, KUNS-1443 HE(TH)97/06, `hep-lat/9705006`.

[51] She-Sheng Xue, Phys. Lett. **B381** (1996) 277; Nucl.Phys. **B486** (1997) 282-314; Nucl. Phys. **B**(Proc.Suppl.) **53** (1997) 668; Phys. Lett. **B395** (1997) 257; MITH-96/10 September 1996, `hep-lat/9609037`; MITH-97/10 June 1996, `hep-lat/9706022`.

[52] L. Maiani, G.-C. Rossi and M. Testa, Phys. Lett. **B292** (1992) 397.

[53] S. Aoki, Int. J. Mod. Phys. **A8** (1993) 5629.

[54] W. Bock, J.E. Hetrick and J. Smit, Nucl. Phys. **B437** (1995) 585.

[55] T. Reisz, Commun. Math. Phys. **116** (1988) 81; M. Lüscher, Selected Topics in Lattice Field Theory, in *Fields, strings and critical phenomena*, E. Brézin and J. Zinn-Justin, eds., Les Houches session XLV (North-Holland, Amsterdam).